IPMUFMF 2025-05-13

[Konvektske množice]

$A \subseteq \mathbb{R}^n$ je konvektna $\Leftrightarrow \forall x, y \in A$, $\lambda \in [0,1]: \lambda x + (1-\lambda) y \in A$

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katere od spodnjih množic so konvektne?

$A = \{1, -1\}$ $\quad B = [-1, 1]$ $\quad C = \{(x,y) \in \mathbb{R}^2 ;\ y \geq x^2\}$

$D = \{(x,y) \in \mathbb{R}^2 ;\ y \leq x^2\}$

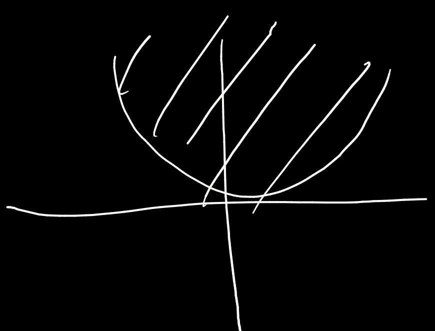
A: protiprimer 0

B: let $x, y \in B$. $\forall \lambda$: $\underline{\lambda x} + \underline{(1-\lambda)} \underline{y \in B}$

$$\underset{\substack{\vee| \\ -1}}{\lambda x} + (1-\lambda) \underset{\substack{\vee| \\ -1}}{y} \geq -\lambda + (1-\lambda) \cdot (-1) = -\lambda - 1 + \lambda = -1$$

$$\lambda x + (1-\lambda) y \leq \lambda + 1 - \lambda = 1$$

$$\Rightarrow \quad \lambda x + (1-\lambda) y \in [-1, 1]$$

C: 

let $T_1, T_2 \in C$

pokažimo $\lambda T_1 + (1-\lambda) T_2 \in C$

$T_1 (x_1, y_1) \in C \Rightarrow y_1 \geq x_1^2$

$T_2 (x_2, y_2) \in C \Rightarrow y_2 \geq x_2^2$

$$\lambda T_1 + (1-\lambda) T_2 = \lambda(x_1, y_1) + (1-\lambda)(x_2, y_2) =$$

$$= (\lambda x_1, \lambda y_1) + ((1-\lambda)x_2, (1-\lambda) y_2) =$$

$$= (\lambda x_1 + (1-\lambda)x_2, \lambda y_1 + (1-\lambda) y_2) \in C$$

$$\lambda y_1 + (1-\lambda) y_2 \geq (\lambda x_1 + (1-\lambda)x^2)^2 =$$

$$= \lambda^2 x_1^2 + 2(1-\lambda)\lambda x_1 x_2 + (1-\lambda)^2 x_2^2 =$$

$$= \dots =$$

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Let $A: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ linearna preslikava in $K \subseteq \mathbb{R}^n$ ter $S \subseteq \mathbb{R}^m$ konveksni. Pokažite, da $A_*(K)$ in $A^*(S)$ konveksni.

$\underline{K \text{ konv} \Rightarrow A_*(K) \text{ konv}}$

$x, y \in A_*(K)$, $\lambda \in [0,1]$, $\underline{\lambda x + (1-\lambda) y \in A_*(K)}$

$\exists a \in K \ni: A(a) = x$

$\exists b \in K \ni: A(b) = y$

$$\lambda A(a) + (1-\lambda) A(b) =$$

$$= A(\lambda a + (1-\lambda) b) \in A_*(K)$$

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$\Omega \subseteq \mathbb{R}^n$ konveksna in $g: \Omega \to \mathbb{R}$ konveksna fja. Pokažite, da je potem $D = \{x \in \Omega; g(x) \leq 0\}$ tudi konveksna.

$\forall x, y \in D, \lambda \in [0,1] \Rightarrow \lambda x + (1-\lambda) y \in D$

$g(x) \leq 0$
$g(y) \leq 0$

ali velja
$g(\lambda x + (1-\lambda) y) \leq 0$?

ker g konveksna,

$$g(\lambda x + (1-\lambda) y) \leq \underbrace{\underbrace{\lambda}_{\geq 0} \underbrace{g(x)}_{\leq 0}}_{\leq 0} + \underbrace{\underbrace{(1-\lambda)}_{\geq 0} \underbrace{g(x)}_{\leq 0}}_{\leq 0} \leq 0$$

naslednjič sem jaz na vrsti: