N

$V = \mathbb{R}_n[x]$. Pokaži, da je $\langle p, q\rangle := \sum_{k=1}^{n+1} p_k q_k$ skalarni produkt in poišči kakšno OB.

skalarni produkt je preslikava : $V^2 \to F$

1.) $\langle u, v\rangle \geq 0$ $\quad \sum_{k=1}^{n+1} p^2 k \geq 0$

$\langle v, v\rangle = 0 \Rightarrow v = 0$ $\quad \sum_{k=1}^{n+1} p_k p_k = 0 \Leftrightarrow \forall k: p_k^2 = 0 \Leftrightarrow \forall k: p_k = 0$

$n+1$ členov, $k \Leftrightarrow p = 0$
toda polinomi so stopnje ≤ 1

2.) $\langle v, u\rangle = \langle u, v\rangle$

$\langle p, q\rangle = \sum_{k=1}^{n+1} p_k q_k = \sum_{k=1}^{n+1} q_k p_k = \langle q, p\rangle$

3.) $\langle \alpha u + \beta v, w\rangle = \alpha\langle u, w\rangle + \beta\langle v, w\rangle$,

$\langle \alpha p + \beta q, r\rangle = \sum_{k=1}^{n+1}(\alpha p_k + \beta q_k) r_k =$

$= \alpha \sum_{k=1}^{n+1} p_k r_k + \beta \sum_{k=1}^{n+1} q_k r_k$

najdi OB (ort. bazo):
iščemo množico $\{p_1, \dots, p_{n+1}\}$ $\ni$: $\langle p_i, p_j\rangle = 0$ $\forall i, j, i \neq j$

$\sum_{k=1}^{n+1} p_{ik} p_{jk}$

N

Definiraj skalarni produkt v $\mathbb{R}_2[x]$, da bodo $1, x-1, 1-x^2$ ONB.
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \| \quad \| \quad \|$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad p_1 \;\; p_2 \;\; p_3$

$p(x) = a + b x^1 + c x^2$
$= a(-p_3 + p_1) + b(p_2 + p_1) + c p_1 =$
$= -a p_3 + b p_2 + (a+b+c) p_1$

$q(x) = d + e x^1 + f x^2 =$
$= -d p_3 + e p_2 + (d+e+f) p_1$

zapišimo std. bazo polinomov po naši ONB:
$1 = p_1$
$x = p_2 + p_1$
$x^2 = p_3 + p_1$

$\langle p, q\rangle = ? = \langle -a p_3 + b p_2 + (a+b+c) p_1, -d p_3 + e p_2 + (d+e+f) p_1\rangle =$

$= ad\langle p_3, p_3\rangle - ae\langle p_3, p_2\rangle - a(a+b+c)\langle p_3, p_1\rangle + be\langle p_2, p_2\rangle + 0 + 0 =$

$+ (a+b+c)(d+e+f)\langle p_1, p_1\rangle = ad + be + (a+b+c)(d+e+f) =$

$= ad + be + (a+b+c)(d+e+f)$

$\hookrightarrow$ to je torej naš skalarni produkt

N

Poišči ONB za $\text{Lin}\{(1,-1,1,1), (-1,1,0,1), (0,0,1,2)\}$

$\langle x_1, \dots, x_n; y_1, \dots, y_n\rangle = \sum_{i=1}^{n} x_i y_i$

Gram - Schmidt:

$$f_1 = v_1$$
$$f_{k+1} = v_{k+1} - \sum_{i=1}^{k} \frac{\langle v_{k+1}, f_i\rangle}{\langle f_i, f_i\rangle} \cdot f_i$$

$$f_1 = (1,-1,1,1)$$
$$f_2 = (1,1,0,1) - \frac{\langle (1,1,0,1), f_1\rangle}{\langle f_1, f_1\rangle} f_1 =$$
$$= (1,1,0,1) + \frac{1}{4}(1,-1,1,1) = \frac{1}{4}(-3,3,1,5)$$

$\tilde{f}_2 = (-3,3,1,5)$ je vzporeden f_2 in ima lepše števile

$$f_3 = (0,0,1,2) - \frac{\langle v_3, f_1\rangle}{\langle f_1, f_1\rangle} f_1 - \frac{\langle v_3, \tilde{f}_2\rangle}{\langle \tilde{f}_2, \tilde{f}_2\rangle} \tilde{f}_2 =$$
$$= (0,0,1,2) - \frac{3}{4}(1,-1,1,1) - \frac{11}{44}(-3,3,1,5) =$$
$$= (0,0,0,0)$$

naš prostor je 2-razsežen.

$$OB: \{(1,-1,1,1), (-3,3,1,5)\}$$

$$ONB: \left\{\frac{f_1}{\|f_1\|}, \frac{f_2}{\|f_2\|}\right\} = \left\{\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \frac{\sqrt{11}}{22}(-3,3,1,5)\right\}$$

N: let $R = \mathrm{Lin}\{(-5,7,0,1), (0,-1,1,0)\}$

a.) $p: \mathbb{R}^4 \to \mathbb{R}^4$ pravokotna projekcija na R.
določi matriko p v std. bazi

$$px = \sum_{i=1}^{n} \frac{\langle x, f_i\rangle}{\langle f_i, f_i\rangle} f_i$$ je projektor na prostor f, kjer je $\{f_i\}_i$ OB f.

$$f_1 = v_1 = (0,-1,1,0)$$
$$f_2 = (-5,7,0,1) - \frac{\langle (-5,7,0,1), (0,-1,1,0)\rangle}{\langle (0,-1,1,0), (0,-1,1,0)\rangle}(0,-1,1,0) =$$
$$= (-5,7,0,1) + \frac{7}{2}(0,-1,1,0) =$$
$$= \left(-5, \frac{7}{2}, \frac{7}{2}, 1\right) \qquad \tilde{f}_2 = (-10,7,7,2)$$

$$p(e_4) = \frac{\langle (0,0,0,1), (0,-1,1,0)\rangle}{\langle f_1, f_1\rangle} f_1 + \frac{\langle (0,0,0,1), (-10,7,7,2)\rangle}{\langle \tilde{f}_2, \tilde{f}_2\rangle} =$$
$$= 0 + \frac{2}{100+49+49+4} \cdot (-10,7,7,2) = \frac{2}{202}(-10,7,7,2)$$

$$p(e_1) = \frac{\langle (1,0,0,0), (0,-1,1,0)\rangle}{\langle f_1, f_1\rangle} f_1 + \frac{\langle (1,0,0,0), (-10,7,7,2)\rangle}{\langle \tilde{f}_2, \tilde{f}_2\rangle} \tilde{f}_2 = \frac{-10}{202}(-10,7,7,2)$$

$$p(e_2) = \frac{\langle (0,1,0,0), (0,-1,1,0)\rangle}{\langle f_1, f_1\rangle} f_1 + \frac{\langle (0,1,0,0), (-10,7,7,2)\rangle}{\langle \tilde{f}_2, \tilde{f}_2\rangle} \cdot \tilde{f}_2 = \frac{7}{202}(-10,7,7,2) - \frac{1}{2}(0,-1,1,0)$$

$$p(e_3) = \frac{\langle (0,0,1,0), (0,-1,1,0)\rangle}{\langle f_1, f_1\rangle} f_1 + \frac{\langle (0,0,1,0), (-10,7,7,2)\rangle}{\langle \tilde{f}_2, \tilde{f}_2\rangle} \cdot \tilde{f}_2 = \frac{1}{2}(0,-1,1,0) + \frac{7}{202}(-10,7,7,2)$$

N

Določi bazo ortogonalnega komplementa R v $R^\perp$.

$$R = \mathrm{Lin}\{\underbrace{(-5,7,0,1)}_{v_1}, \underbrace{(0,-1,1,0)}_{v_2}\}$$

$$R^\perp = \{\vec{x} \;;\; \langle \vec{x}, v_i\rangle, \forall i \in \{1,2\}\}$$

$$\langle (x,y,z,w), (-5,7,0,1)\rangle =$$

$$= -5x + 7y + w$$

$$\langle (x,y,z,w), (0,-1,1,0)\rangle = z - y$$

$$z - y = 0$$
$$z = y$$

$$-5x + 7z + w = 0$$
$$w = 5x - 7y$$

$$(0, 1, 1, -7)$$
$$(1, 0, 0, 5)$$

N

Poišči OB za $\mathbb{R}_3[x]$ glede na $\langle f, g\rangle = \int_{-1}^{1} f t g t dt$

ena baza: $\{1, x, x^2, x^3\}$

$$\langle 1, 1\rangle = \int_{-1}^{1} 1 \, dx = (1+1)\cdot 1 = 2$$

$$\langle x, 1\rangle = \int_{-1}^{1} x \, dx = 0$$

$$\langle x^2, 1\rangle = \int_{-1}^{1} t^2 \, dt = \frac{t^3}{3}\Big|_{-1}^{1} = \frac{2}{3}$$

$$\langle x^2, x\rangle = \int_{-1}^{1} t^3 dt = 0 \quad \text{(liha)}$$

$$f_1 = 1$$
$$f_2 = x$$
$$f_3 = x^2 - \frac{\langle x^2, 1\rangle}{\langle 1, 1\rangle}\cdot 1 - \frac{\langle x^2, x\rangle}{\langle x, x\rangle}\cdot x = x^2 - \frac{1}{3}$$

$$f_4 = x^3 - \frac{\langle x^3, 1\rangle}{\langle 1, 1\rangle}\cdot 1 - \frac{\langle x^3, x\rangle}{\langle x, x\rangle}x - \frac{\langle x^3, x^2 - 1/3\rangle}{\langle x^2 - \frac{1}{3}, x^2 - \frac{1}{3}\rangle}\left(x^2 - 1/3\right) =$$

0 lichost 0 lichost

$$= -\frac{\langle x^3, x\rangle}{\langle x, x\rangle}x = \boxed{x^3 - \frac{3}{5}x}$$

$$\langle x^3, x\rangle = \int_{-1}^{1} x^4\,dx = \frac{x^5}{5}\Big|_{-1}^{1} = \frac{1}{5} - \left(-\frac{1}{5}\right) = \frac{2}{5}$$

$$\langle x, x\rangle = \int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{2}{3}$$