$$A=\left[\begin{array}{ccc|c|cc} 1 & 2 & 2 & 0 & & \\ 2 & -10 & -7 & 0 & & 0 \\ -3 & 17 & 12 & 0 & & \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ \hline & 0 & & 0 & -5 & 9 \\ & & & 0 & -4 & 7 \end{array}\right]$$

A bločno diagonalna.

$$\Longrightarrow \quad J(A)=\begin{bmatrix} J(A_1) & & \\ & J(A_2) & \\ & & J(A_3) \end{bmatrix}$$

$$P(A)=\begin{bmatrix} P(A_1) & & \\ & P(A_2) & \\ & & P(A_3) \end{bmatrix}$$

$$A_1=\begin{bmatrix} 1 & 2 & 2 \\ 2 & -10 & -7 \\ -3 & 17 & 12 \end{bmatrix}$$

$$A_2=\begin{bmatrix} -5 & 9 \\ -4 & 7 \end{bmatrix}$$

$$\Delta_{A_1}=\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & -10-\lambda & -7 \\ -3 & 17 & 12-\lambda \end{vmatrix} = \dots \text{DN.} = (1-\lambda)^3$$

$m_A(x)$: kandidati $(1-\lambda)^3,\ (1-\lambda)^2,\ (1-\lambda)$.

in $m_A(A)=0$:

$m_A(x)=1-\lambda$: $\begin{bmatrix} 0 & -2 & -2 \\ -2 & 11 & 7 \\ 3 & -17 & -11 \end{bmatrix}$ CRK

$$m_A(x)=(1-\lambda)^2=\begin{bmatrix} 0 & -2 & -2 \\ -2 & 11 & 7 \\ 3 & -17 & -11 \end{bmatrix}^2 = \dots \neq 0 \quad \text{CRK}$$

torej $m_A(x)=(1-\lambda)^3$. $\rightarrow$ stopnja v m_A je velikost najv. celice.

$$J(A_1)=\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$P(A_1)=\ ?$ iščemo v, ki je $\in \ker(B^3)$ in $\notin \ker(B^2)$

za $B=A_1-\lambda I=A_1-I$

$v \in \underbrace{\ker B^3}_{\text{vse}} \setminus \ker B^2$

B^3 je ničelna matrika

$$P(A_1)=\begin{bmatrix} -2 & 0 & 1 \\ -1 & 2 & 0 \\ 1 & -3 & 0 \end{bmatrix}$$

tak v je recimo $(1,0,0)$

in $Bv=\begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$ in $B^2v=\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$

$J(A_2)=[0] \qquad P(A_2)=[1]$

za 2×2 matrike:

- sled in det dveh podobnih matrik je enaka
- $\left.\begin{array}{l} \text{sled } A_3=\lambda_1+\lambda_2=-5+7=2 \\ \det A_3=\lambda_1\lambda_2=-35+36=1 \end{array}\right\}\ \lambda_1=\lambda_2=1$

$m_{A_3}(\lambda)=(\lambda-1)^2$, max celica je 2×2

$J(A_3)=\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. $\quad v \in \ker B^2 \setminus \ker B$, $\quad B=\begin{bmatrix} -6 & 9 \\ -4 & 6 \end{bmatrix}$, $\quad v=\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\quad v_2=Bv=\begin{bmatrix} -6 \\ -4 \end{bmatrix}$

$$P(A_3) = \begin{bmatrix} -6 & 1 \\ -4 & 0 \end{bmatrix} \qquad \text{torej}$$

$$J(A) = \begin{bmatrix} 1 & 1 & 0 & 0 & & \\ 0 & 1 & 1 & 0 & & 0 \\ 0 & 0 & 1 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & & 0 & 1 & 1 \\ & & & 0 & 0 & 1 \end{bmatrix} \qquad P(A) = \begin{bmatrix} -2 & 0 & 1 & 0 & & \\ -1 & 2 & 0 & 0 & & 0 \\ 1 & -3 & 0 & 0 & & \\ 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 1 & 1 \\ & 0 & & 0 & 0 & 1 \end{bmatrix}$$

---

kako dobiti št. kletk A_1 z drugim postopkom:

$\dim \ker B = \dots$

$$\underbrace{\dim \operatorname{Im} B}_{\text{z gaussom: } 2} + \underbrace{\dim \ker B}_{\to\ 1\ \checkmark} = \dim B = 3$$

N

$$A = \begin{bmatrix} 2 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 4 & 1 \\ 0 & 0 & -1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

A_1, A_2, A_3

Bločno zgornje trikotna. Izračunaj ρ in J_A.

$$\det(A-\lambda I) = \det(A_1-\lambda I)\det(A_2-\lambda I)\det(A_3-\lambda I)$$

$$\left.\begin{array}{l} \operatorname{sl} A_1 = 2 = \lambda_1+\lambda_2 \\ \det A_1 = 1 = \lambda_1 \lambda_2 \end{array}\right\} \lambda_1 = \lambda_2 = 1$$

$$\left.\begin{array}{l} \operatorname{sl} A_2 = 2 = \lambda_1'+\lambda_2' \\ \det A_2 = 1 = \lambda_1' \lambda_2' \end{array}\right\} \lambda_1' = \lambda_2' = 1$$

$$\left.\begin{array}{l} \operatorname{sl} A_3 = 1 = \lambda_1'' \\ \det A_3 = 1 = \lambda_1'' \end{array}\right\} \lambda_1'' = 1$$

$$J_A = \begin{bmatrix} 1 & ? & & & \\ & 1 & ? & ? & \\ & & 1 & ? & \\ & 0 & & 1 & \\ & & & & 1 \end{bmatrix}$$

kaj pa število celic?

$m_A = (\lambda-1)^r$ iščemo rang $A-\lambda$

$$A-\lambda = B = r\begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & -2 & 4 & 1 \\ 0 & 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \underset{\text{gauss}}{=} r\begin{bmatrix} \dots \end{bmatrix} = 3$$

$\Rightarrow \dim \ker B = 2.$ $\quad B^2 = \begin{bmatrix} 0 & & 0 \\ & & 2 \\ & & 1 \\ & & 0 \end{bmatrix} \to B^3 = 0$

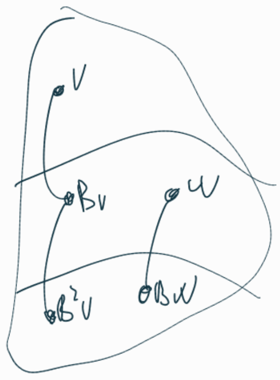
$v \in \underbrace{\ker B^3}_{\text{vse}} \setminus \ker B^2$

$v = e_5$: $Bv = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ $B^2 v = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$

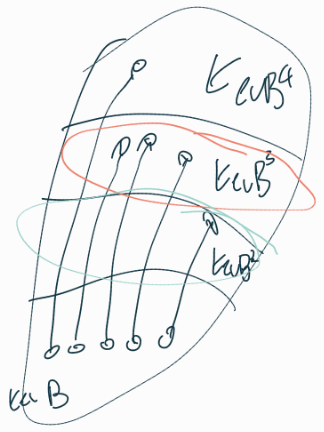
$w \in \ker B^2 \setminus \ker B$, w LN z Bv.

Vzemimo e_1, $Bw = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$$P = \begin{bmatrix} 0 & -1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$



število kletk velikosti $\geq k$:



$\dim \ker B^k - \dim \ker B^{k-1} =$ št. kletk velikosti $\geq k$

$m - m = 4$ $\quad \geq 4$

N

Let $A \in \mathbb{C}^{11 \times 11}$

$\dim \ker A^4 = 11$
$\dim \ker A^3 = 10$
$\dim \ker A^2 = 7$

izračunaj $\dim \ker A^1$ ....?!

N

Reši sistem linearnih rekurzivnih enačb:

$x_{n+1} = -4x_n + 4y_n$

$y_{n+1} = -x_n$

$x_0 = 0 \quad y_0 = 1$

Vzemimo tak A, da $A \begin{bmatrix} x_n \\ y_n \end{bmatrix} = \begin{bmatrix} x_{n+1} \\ y_{n+1} \end{bmatrix}$

$$A = \begin{bmatrix} -4 & 4 \\ -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_n \\ y_n \end{bmatrix} = A \begin{bmatrix} x_{n-1} \\ y_{n-1} \end{bmatrix} = A^n \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = A^n \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} -4 & 4 \\ -1 & 0 \end{bmatrix}^n}_{\text{z Jordansko formo}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\Delta_{\begin{bmatrix} -4 & 4 \\ -1 & 0 \end{bmatrix}}(\lambda) = \det(A - \lambda I) = \begin{vmatrix} -(4+\lambda) & 4 \\ -1 & -\lambda \end{vmatrix} = \lambda(4+\lambda) + 4 = 4\lambda + \lambda^2 + 4 = (\lambda+2)^2$$

$\lambda = -2$

kandid: $\{1, 2\}$

Toda $m_A(\lambda) = (\lambda+2)^2$

max klet. vel 2

$$J_A = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}$$

$v = \ker B^2 \setminus \ker B$.

$B = A + 2I = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$

$v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $Bv = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$; $P = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix}$

$$P^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$$

$$A^n = P J_A^n P^{-1} = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix}^n}_{} \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} =$$

$$\begin{bmatrix} (-2)^n & n(-2)^{n-1} \\ 0 & (-2)^n \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} (-2)^n (n+1) & (-2^{n+1}) n \\ -n(-2)^{n-1} & (-2)^n (1-n) \end{bmatrix}}_{}$$

množimo z $\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \dots \quad \begin{bmatrix} (-2^{n+1}) n \\ (-2)^n (1-n) \end{bmatrix} =$

$$(-2)^n \begin{bmatrix} -2n \\ 1-n \end{bmatrix}$$

N

$\det e^A = e^{\operatorname{sled} A}$ napotek: $e^A = P e^{J_A} P^{-1}$

$$\det e^A = \det(P e^{J_A} P^{-1}) = \det P \det e^{J_A} \det P^{-1} =$$

$$= \cancel{\det P \det P^{-1}} \det e^{J_A}$$

$$J_A = \begin{bmatrix} \lambda_1 & * & & 0 \\ & \ddots & \ddots & \\ & & \ddots & * \\ 0 & & & \lambda_n \end{bmatrix} \qquad e^{J_A} = \begin{bmatrix} e^{\lambda_1} & & * \\ & \ddots & \\ 0 & & e^{\lambda_n} \end{bmatrix}$$

$$\det e^{J_A} = e^{\lambda_1} \cdots e^{\lambda_n} = e^{\lambda_1 + \dots + \lambda_n} = e^{\operatorname{sled} A}$$