LAINFMF 2024-02-27

c) $U+V = U \cup V \Leftrightarrow (U \subseteq V \lor V \subseteq U)$

($\Rightarrow$): $U+V = U \cup V$ predpostavka

case 1: recimo, da ne velja $U \subseteq V$.

$U \not\subseteq V$. $\underline{V \subseteq U}$

vzemimo poljuben $v \in V$. dokažimo $v \in U$

vzemimo $u \in U$. vemo $u+v \in U+V$ (očitno)

$\Rightarrow u+v \in U \cup V$

$\Rightarrow \underline{u+v \in U} \lor u+v \in V$

upoštevamo $U \not\subseteq V$:

$\exists u' \in U \setminus V$.

recimo $u'+v \in V$.

$\underbrace{(u'+v)}_{\in V} - \underbrace{v}_{\in V} = u' \notin V$

$\times$!

torej $u'+v \in U$

$v \in U$

$u'+v \in U$

$u'+v-u' \in U$

$\underline{v \in U}$

($\Leftarrow$): predpostavka: $U \subseteq V \lor V \subseteq U$

dokazujemo $\underline{U+V \subseteq U \cup V} \land \underline{U \cup V \subseteq U+V}$

↳ očitno,

ker

$U+V = \{u+v;\ u \in U, v \in V\}$

$U+V \subseteq U \cup V$

Primeri:

case $U \subseteq V$: (glej a) iz nečinih ~~zapisov~~ predfinje velje)

$U+V = V$

$\Downarrow$ $\Rightarrow U+V = V = U \cup V$

$U \cup V = V$

case $V \subseteq U$: $U+V = U$, $U \cup V = U$ $\Rightarrow U+V = U = U \cup V$

d) $U \cup V$ je vektorski podprostor $\Leftrightarrow U \subseteq V \lor V \subseteq U$

($\Rightarrow$): predpostavimo $U \cup V$ je vektorski podprostor:

$U \cup V \subseteq U+V$

najmanjši vektorski prostor, v katerem sta U in V, je $U+V$.

$\Rightarrow U \cup V = U+V$

($\Leftarrow$): predpostavimo $U \subseteq V$ v $V \subseteq U$

$U \cup V$ je vektorski podprostor ...

„težavnej: $V \cup U$ ni vedno vektorski podprostor"

N

Določi, katere izmed preslikav so linearne!

linearne preslikave so homomorfizmi vektorskih prostorov.

$A: V \to V$ je linearna $\Leftrightarrow$ homogena in aditivna

$$A(\alpha x + \beta y) = \alpha A(x) + \beta A(y)$$

a) $A: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad A(x,y) = (x+1, y-1)$

leva stran: $A(\alpha(x_1, y_1) + \beta(x_2, y_2)) = A((x_1\alpha, y_1\alpha) + (x_2\beta, y_2\beta)) =$

$= A((x_1\alpha + x_2\beta, y_1\alpha + y_2\beta)) = (x_1\alpha + x_2\beta + 1, y_1\alpha + y_2\beta - 1)$

desna stran: $\alpha A(x_1, y_1) + \beta A(x_2, y_2) = \alpha(x_1+1, y_1-1) + \beta(x_2+1, y_2-1) =$

$= (\alpha(x_1+1), \alpha(y_1-1)) + (\beta(x_2+1), \beta(y_2-1)) =$

$= (\alpha x_1 + \alpha + \beta x_2 + \beta,\ \alpha y_1 - \alpha + \beta y_2 - \beta)$ ni L.P.

ni enako DRE!

ali pa preverimo $A(0,0) \neq (0,0)$

N

$B: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad B(x,y) = (2x+y, y-x)$

... je!

N

$\checkmark\checkmark\checkmark$

N

a) $A: \mathbb{R}^3 \to \mathbb{R}^3$

$A(x,y,z) = (x+y-z, x+z, 2x+y)$

L: $A(\alpha(x_1, y_1, z_1) + \beta(x_2, y_2, z_2)) =$

$= A((\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2, \alpha z_1 + \beta z_2)) =$

$= (\alpha x_1 + \beta x_2 + \alpha y_1 + \beta y_2 - \alpha z_1 - \beta z_2,$

$\alpha x_1 + \beta x_2 + \alpha z_1 + \beta z_2,$

$2\alpha x_1 + 2\beta x_2 + \alpha y_1 + \beta y_2)$

D: $\alpha A(x_1, y_1, z_1) + \beta A(x_2, y_2, z_2) =$

$= \alpha(x_1+y_1-z_1, x_1+z_1, 2x_1+y_1) + \beta(x_2+y_2-z_2, x_2+z_2, 2x_2+y_2) = \ldots =$ isto

dn. st. ne bo.

N

b.) primer preizkusne naloge

zapiši matriko preslikave A v standardni bazi

$$A(0,0,1) = (-1,1,0)$$
$$A(0,1,0) = (1,0,1)$$
$$A(1,0,0) = (1,1,2)$$

$$A_S = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

S tot standardna baza

c) zapiši matriko preslikave A v bazi:

$$B = \{\underset{\vec{a}}{(1,0,0)},\ \underset{\vec{b}}{(1,1,0)},\ \underset{\vec{c}}{(1,1,1)}\}$$

slike:

$$\{(1,1,2),\ (2,1,3),\ (1,2,3)\}$$

$$(1,1,2) = -\vec{b} + 2\vec{c}$$
$$(2,1,3) = \vec{a} - 2\vec{b} + 3\vec{c}$$
$$(1,2,3) = -\vec{a} - \vec{b} + 3\vec{c}$$

$$A_B = \begin{bmatrix} 0 & 1 & -1 \\ -1 & -2 & -1 \\ 2 & 3 & 3 \end{bmatrix}$$

d.) zapiši prehodni matriki

$$P_{S\leftarrow B} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

velja: $P_{A\leftarrow B} = (P_{B\leftarrow A})^{-1}$!

$$P_{B\leftarrow S} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

in preveri, da je $A_B = P_{B\leftarrow S} \cdot A_S \cdot P_{S\leftarrow B}$

drži.

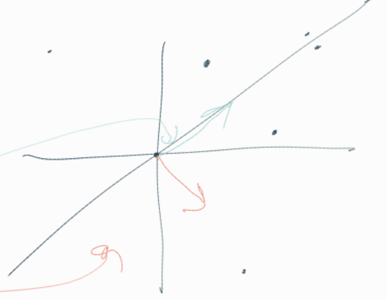
N

zapiši matriko v standardni bazi, ki pripada zrcaljenju čez premico

$$y = tx$$

$$A(x, tx) = (x, tx)$$
$$A(1,t) = (1,t)$$
$$A(t,-1) = -(t,-1)$$

baza



$$A_B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad B = \left\{\begin{bmatrix} 1 \\ t \end{bmatrix}, \begin{bmatrix} t \\ -1 \end{bmatrix}\right\}$$

$$P_{S\leftarrow B} = \begin{bmatrix} 1 & t \\ t & -1 \end{bmatrix}$$

$$A_S = P_{S\leftarrow B}\, A_B\, P_{B\leftarrow S} =$$
$$= P_{S\leftarrow B}\, A_B\, (P_{S\leftarrow B})^{-1} = \dots$$