Izračunaj det:

$$D_n = \begin{vmatrix} a & & & & & b \\ & a & & & b & \\ & & a & b & & \\ & & b & a & & \\ & b & & & a & \\ b & & & & & a \end{vmatrix} \Bigg\} 2n$$

$$\underbrace{\qquad\qquad\qquad}_{2n}$$

$$= a\cdot(-1)^{i+j}\begin{vmatrix} a & & & & b & 0 \\ & \ddots & & \iddots & & \\ & & ab & & & \\ & & ba & & & \\ & \iddots & & \ddots & & \\ b & & & & a & a \end{vmatrix} + b\cdot(-1)^{i+j}\begin{vmatrix} a & & & & & b \\ & \ddots & & \iddots & & \\ & & ab & & & \\ & & ba & & & \\ & \iddots & & \ddots & & \\ b & & & & a & 0 \end{vmatrix} =$$

$$= a\cdot(-1)^{2n-1+2n-1} D_{n-1} +$$

$$+ b\cdot(-1)^{1+2n-1} D_{n-1} = \dots$$

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$$\begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & 1-x & & & 1 \\ \vdots & & 2-x & & \vdots \\ 1 & & & (n-1-x) & 1 \\ 1 & 1 & \cdots & 1 & (n-x) \end{vmatrix} \xrightarrow{V_t' = V_t - V_1} \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & -x & & & 0 \\ & & 1-x & & \\ & & & 2-x & \\ 0 & & & & 0 \\ 0 & & & 0 & n-1-x \end{vmatrix} = 1\cdot(-x)\cdot(1-x)\cdots(n-1-x)$$

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$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ t+1 & 2 & t+3 & 4 \\ 1 & t+3 & t+4 & t+5 \\ 1 & -3 & -4 & -5 \end{vmatrix} \overset{\substack{II-=I \\ III-=I \\ IV-=I}}{=\!=} \begin{vmatrix} 1 & 2 & 3 & 4 \\ t & 0 & t & 0 \\ 0 & t+1 & t+1 & t+1 \\ 0 & -5 & -7 & -9 \end{vmatrix} = t(t+1)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -7 & -9 \end{vmatrix} \overset{II-=I}{=\!=} t(t+1)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -2 & -4 \\ 0 & 1 & 1 & 1 \\ 0 & -5 & -7 & -9 \end{vmatrix} \underset{IV+5III}{\overset{II+=I}{=\!=}}$$

$$t(t+1)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -4 \end{vmatrix} \underset{IV+=2II}{\overset{III-=II}{=\!=}} t(t+1)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & -4 \end{vmatrix} = -t(t+1)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -4 \end{vmatrix} = 4t(t+1)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} =$$

$$\overset{II-=IV}{=\!=} \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 4t(t+1) = 4t^2 + 4$$

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$A, B, C \quad \mathbb{R}^{5\times 5} \qquad \det A = 2 \quad \det B = 3 \quad \det C = 0$

a) $\det(AB^T) = ? = \det A \det B^T = \det A \det B = 2\cdot 3 = 6$

b) $\det(A^{-1}B^3) = \det A^{-1}(\det B)^3 = 27 \det A^{-1} = 27/2$

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$$\det\left(3A^2\left(\frac{B^TB}{2}\right)^{-1}\right) = \det(3A^2)\left(\det\frac{B^TB}{2}\right)^{-1} = 3^5\cdot 9\left(\frac{1}{2^5}\det B^TB\right)^{-1} =$$

$$= 3^5 \cdot 9 \left(\frac{1}{2^5} \det B \cdot \det B^{-1}\right) = 3^5 \cdot 9 \left(\frac{1}{2^5} \cdot 4\right)^{-1} = 3^5 \cdot 9 \cdot \frac{32}{4} =$$

$$= 3^5 \cdot 9 \cdot 8 = 3^5 \cdot 72 = 72 \cdot 81 \cdot 3 \quad ??$$

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$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix} = E_{2,1}\left(-CA^{-1}\right)^{-1} \begin{bmatrix} A & 0 \\ 0 & M/A \end{bmatrix} E_{1,2}\left(-1^{-1}B\right)^{-1}$$

$$\begin{vmatrix} A & B \\ C & 0 \end{vmatrix} = \det \begin{bmatrix} A & 0 \\ 0 & M/A \end{bmatrix} = \det A \det M/A$$

$$= \det A \det\left(0 - CA^{-1}B\right)$$

če $\exists A^{-1}$

Ali če $\exists D^{-1}$:

$\det A \det M/$

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S cramerjevim pravilom reši

$$\begin{aligned} x + y - z &= 6 \\ 3x - 2y + z &= -5 \\ x + 3y - 2z &= 14 \end{aligned}$$

če $\det A \neq 0$

$$x_i = \frac{\det A_i(b)}{\det A}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & -2 & 1 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 14 \end{bmatrix}$$

$$\det A = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -2 & 1 \\ 1 & 3 & -2 \end{vmatrix} \begin{matrix} 1 & 1 \\ 3 & -2 \\ 1 & 3 \end{matrix} = (2+3-6-4-1+9 = 14-11) = 3$$

$$\det A_1 = \begin{vmatrix} 6 & 1 & -1 \\ -5 & -2 & 1 \\ 14 & 3 & -2 \end{vmatrix} \begin{matrix} 6 & 1 \\ -5 & -2 \\ 14 & 3 \end{matrix} = 24+14+15-10-18-28 = \cancel{5}\overset{4}{}3 - \cancel{5}\overset{4}{}6 = -3$$

$$\det A_2 = \begin{vmatrix} 1 & 6 & -1 \\ 3 & -5 & 1 \\ 1 & 14 & -2 \end{vmatrix} \begin{matrix} 1 & 6 \\ 3 & -5 \\ 1 & 14 \end{matrix} = \cancel{10+6-12+36-4-5 = 52-21 = 31}$$

$$\det A_3 = \begin{vmatrix} 1 & 1 & 6 \\ 3 & -2 & -5 \\ 1 & 3 & 14 \end{vmatrix} \begin{matrix} 1 & 1 \\ 3 & -2 \\ 1 & 3 \end{matrix} = \cancel{-6-5+54+12+15-12} =$$

...

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$$A^{-1} = \frac{1}{\det A} \tilde{A}^T$$

→ Zofaktorska matrika $= \{\tilde{a}_{i,j}\}_{ij}$

$$\tilde{a}_{ij} = (-1)^{i+j} \det A_{ij}$$

→ A brez i-te vrstice in j-tega st.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \tilde{A} = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix} \qquad A^{-1} = \frac{1}{4-6} = -\frac{1}{2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} =$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \tilde{A} = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix} \qquad = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}^{-1} = ? \qquad \tilde{B} = \dots$$

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Dokaži, da je $V(a_1, \dots, a_n) = \det \begin{bmatrix} 1 & 1 & \dots & 1 \\ a_1 & \dots & & a_n \\ a_1^2 & \dots & & a_n^2 \\ \vdots & & & \vdots \\ a_1^{n-1} & \dots & & a_n^{n-1} \end{bmatrix} =$

(Vandermondova baza)

$$= \prod_{i>j} (a_i - a_j) \quad \dots$$