KOMB VFMF 2024-12-10

X množice

$G \leq S_X$ G deluje na X

X/G ... množica orbit X pri delovanju G

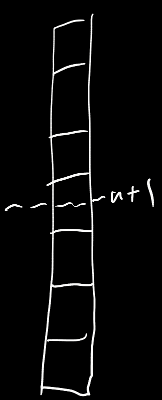
Burnsideva lema

$$|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$$

#orbit

N

$2n+1$ enako visokih polj.



Na koliko načinov lahko drog pobarvamo s tremi barvami, če dve zaporedni polji nista enake barve

$G = \{id, \tau\}$ $\tau = (1\ 2n+1)(2\ 2n) \dots (n\ n+2)(n+1)$

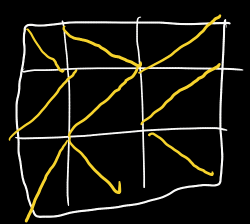
...

N

Ploščice 3x3, vrtimo v ravnini

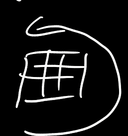
a.) določi grupo G in ciklični indeks

b.) na koliko načinov lahko v kvadratke narišemo diagonale – vsakega natanko eno – do upoštevanja simetrij.



$G = \{id, \rho_{90}, \rho_{90}^2, \rho_{90}^3\}$

ρ_{90} rotacija $90°$



$90°$

$\tau_{90} = (1\,7\,9\,3)(2\,4\,8\,6)(5)$

$\tau_{90}^2 = (1\,9)(7\,3)(1\,8)(4\,6)(5)$

$\tau_{90}^3 = (1\,3\,9\,7)(2\,6\,8\,4)$

$t_G(t_1, \dots, t_9) = \frac{1}{4}\left(t_1^9 + 2t_4^2 t_1 + t_2^4 t_1\right)$

X … označena polja.

id … 2^9 (9 polj, ∀ polje ✓/ali ✗)

$\tilde\tau_{90}$ … 0

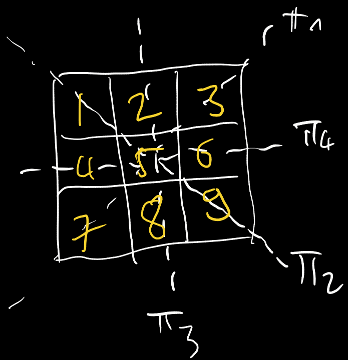
τ_{90} … 0

τ_{180} … 2^5 v vsakem ciklu izberemo diagonalo, drugo je s tem določena.

$$|X/G| = \frac{1}{4}\left(2^9 + 2^5\right) = 2^7 + 2^3 = 128 + 8 = 136$$

Sedaj še obračanja plošče

$H = G \cup \{\pi_1, \pi_2, \pi_3, \pi_4\}$

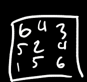


$\pi_1 = (1\,9)(2\,6)(4\,8)(3)(5)(7)$

$\pi_2 = (3\,7)(2\,4)(6\,8)(1)(5)(9)$

$\pi_3 = (1\,3)(4\,6)(7\,9)(2)(5)(8)$

$\pi_4 = (1\,7)(2\,8)(3\,9)(4)(5)(6)$

π_1 $(, \pi_2)$:  $2^6 = 64$

π_3 $(, \pi_4)$: 0

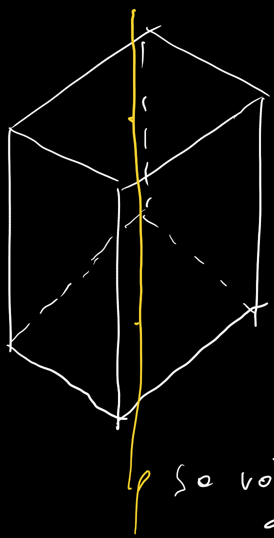
$H \cdot 1 = \{1, 3, 7, 9\}$ (id, τ_{90}, τ_{90}^2, τ_{90}^3)

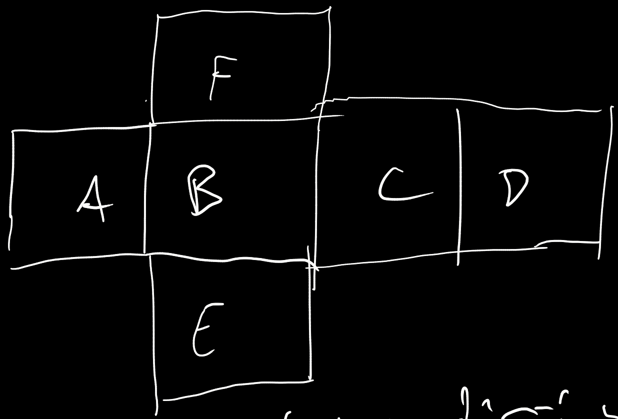
$$|X/H| = \frac{1}{8}\left(\underbrace{2^9 + 2^5}_{\text{iz } G} + 2 \cdot 2^6\right) = 2^6 + 2^2 + 2^4 = 64 + 4 + 16 = 84$$

(iz H)

$H_1 = \{id, \pi_2\}$ $|H| = |H \cdot 1| \cdot |H_1| = 2 \cdot 4 = 8$

N



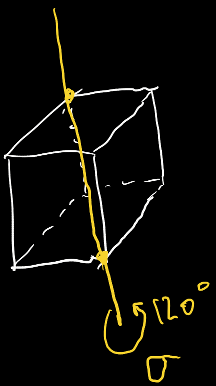


ρ so rotacije okoli osi skozi središči nasprotnih ploskev za 90°.

$\rho = (ABCD)(F)(E)$

$\rho^2 = (AC)(BD)(F)(E)$

$\rho^3 = \rho^{-1} = (DCBA)(E)(F)$

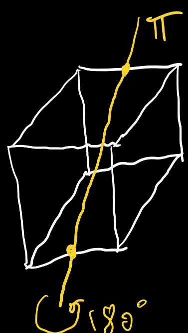


rotacije σ za 120° okrog osi skozi par nasprotnih oglišč.

$\sigma = (ABE)(DFC)$

$\sigma^2 = (EBA)(CFD)$

$\Big\} \times 4$



rotacije π za 180° okrog osi skozi presečišči nasprotnih robov

$\pi = (DB)(FC)(AE)$

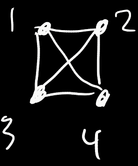
...

N na toliko načinov

...

N grupa S_4 deluje na vozliščih grafa K_4.

$Z_{S_4}(t_1,t_2,t_3,t_4) = \frac{1}{24}(t_1^4 + 6t_1^2t_2^1 + 8t_1^1t_3^1 + 3t_2^2 + 6t_4^1)$



tip	perm	ciklični indeks pri delovanju na vozliščih
id	1	t_1^4
(ab)(c)(d)	$\binom{4}{2}=6$	$t_1^2 t_2^1$
(abc)(d)	$4\cdot 2 = 8$ (↓ fiksna točka, ↓ smer cikla)	$t_3 t_1$
(ab)(cd)	$\binom{4}{2}/2 = 3$	t_2^2
(abcd)	$(4-1)! = 6$	t_4^1
	24	

Drugi del naloge: naj S_4 deluje točno na povezave grafa na naravni način.

$\pi\{u,v\} = \{\pi u, \pi v\}$

Na koliko načinov lahko pobarvamo povezave grafa K_4 ob upoštevanju simetrij?

(↑ z dvema barvama)

barva 1: povezava je

barva 2: povezave ni

...

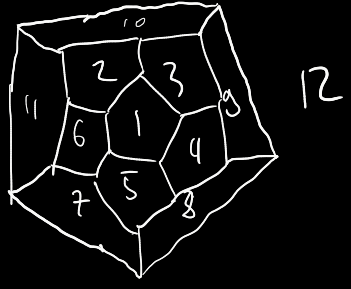
G je grupa simetrij dodekaedra, deluje na ploskvah.

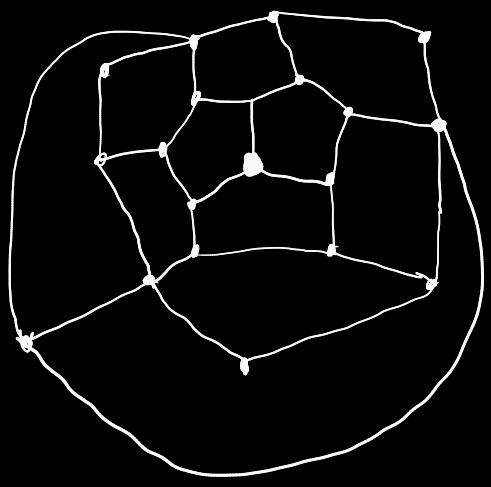
$|G1| = 12$

$|G_1| = 5$ stabilizator

$G_1 = \{id, \rho, \rho^2, \rho^3, \rho^4\}$ $\rho = (1)(23456)(78910)(12)$

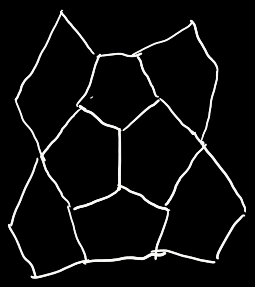
$|G| = 12\cdot 5 = 60$





$\times 10$ $\begin{cases} \pi = (1\ 2\ 6)(3\ 11\ 5)(4\ 7\ 10)(8\ 12\ 9) \\ \pi^2 = \dots \end{cases}$

180° po presek. na pr. str.



$$\tau = (12)(3\ 6)(4\ 11)(5\ 7)(8\ 10)(9\ 12)$$

$$Z_G(t_1, \dots, t_{12}) = \frac{1}{60}(\dots$$