KOMB V FMF 2024-11-12

# Binomski izrek

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$$

$$a(x) = \sum_{i=0}^{n} a_i x^i \qquad c(x) = a(x)b(x) = \sum_{k=0}^{n+m} c_k x^k$$

$$b(x) = \sum_{j=0}^{m} b_j x^j \qquad c_k = \sum_{i=0}^{k} a_i b_{k-i}$$

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Z uporabo binomskega izreka seštej vrsti.

$$\sum_{k=0}^{n} \binom{n}{k} 2^k = (2+1)^n = 3^n$$

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = (1-1)^n = 0^n = \delta_{0n}$$

$$\sum_{k=0}^{n} \binom{n}{k} = (1+1)^n = 2^n$$

$[n]$

Koliko podmnožic sode moči ima množica z n elementi?

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \binom{n}{6} + \dots =$$

$$= \sum_{k=0} \binom{n}{2k} = \left( \sum_{k=0}^{n} (-1)^k \binom{n}{k} + \sum_{k=0}^{n} \binom{n}{k} \right) \cdot \frac{1}{2} = 2^{n-1}$$

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Koliko matrik $A \in \{0, \dots, m\}^{n \times n}$

a.) Nimajo ničelne vrstice

b.) Nimajo ničelnega stolpca niti ničelne vrstice

a.) $(m+1)^n$ ... možne vrstice

$(m+1)^n - 1$ ... možne neničelne vrstice

$((m+1)^n - 1)^n$ ... iskano število

b.) $\sum_{I,J \subseteq [n]} (-1)^{|I|+|J|} (n+1)^{(n-|I|)(n-|J|)}$

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Zaporedju $(a_i)_{i=0}^{\infty}$ priredimo vsoto

$$A(x) = \sum_{i=0}^{N} a_i x^i$$

$$\sum_{i=0}^{\infty} x^i = 1 + x + x^2 + \dots = \frac{1}{x-1} \quad \Rightarrow \quad (1-x)^{-1} = 1 + x + x^2 + \dots$$

$$(1 + x + x^2 + \dots)(1-x) = 1$$

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a.) $1, 0, 1, 0, 1, 0, \dots$

$$A(x) = 1 + x^2 + x^4 = \sum_{i=0}^{\infty} x^{2i} = \frac{1}{1-x^2}$$

b.) $0, 1, 0, 1, 0, 1, \dots$

$$B(x) = x + x^3 + x^5 + \dots = x A(x) = \frac{x}{1-x^2}$$

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$$A(x) = \sum_{n=0}^{\infty} a_n x^n$$

a.) $(p_n) : p_n = 2a_n \qquad P(x) = \sum_{n=0}^{\infty} p_n x^n = \sum_{n=0}^{\infty} 2a_n x^n = 2 \cdot A(x)$

b.) $q_n : a_n + 2 \qquad Q(x) = \sum_{n=0}^{\infty} (a_n + 2) x^n = \sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} 2 x^n =$

$$= A(x) + 2 \frac{1}{1-x}$$

c.) $r_n = a_{n-2}$ za $n \geq 2 \qquad R(x) = \sum_{n=2}^{\infty} a_{n-2} x^n = \sum_{n=0}^{\infty} a_n x^{n+2} =$

$$= \sum_{n=0}^{\infty} a_n x^n \cdot x^2 = x^2 A(x)$$

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Reši rekurzivno enačbo:

$$a_{n+1} - 2a_n = 4^n \quad / \cdot x^{n+1} \quad (n \geq 0), \quad a_0 = 1$$

$$a_{n+1} x^{n+1} - 2a_n x^{n+1} = 4^n x^{n+1} \quad / \sum$$

$$\sum_{n=0}^{\infty} a_{n+1} x^{n+1} - \sum_{n=0}^{\infty} 2a_n x^{n+1} = \sum_{n=0}^{\infty} 4^n x^{n+1}$$

$$A(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$A(x) - \overset{1}{a_0} - 2x A(x) = x \frac{1}{1-4x}$$

$$A(x) = \frac{\frac{x}{1-4x} + 1}{1-2x} = \frac{\frac{1-3x}{1-4x}}{1-2x} = \frac{1-3x}{(1-4x)(1-2x)} =$$

$$= \frac{A}{1-4x} + \frac{B}{1-2x} = \frac{\frac{1}{2}}{1-4x} + \frac{\frac{1}{2}}{1-2x} =$$

$$= \frac{1}{2} \sum_{n=0}^{\infty} (2x)^n + \frac{1}{2} \sum_{n=0}^{\infty} (4x)^n = \sum_{n=0}^{\infty} \left(\frac{1}{2} \cdot 2^n \cdot \frac{1}{2} 4^n\right) x^n =$$

$$= \sum_{n=0}^{\infty} \frac{1}{2} (2^n + 4^n) x^n \qquad a_n = \frac{1}{2}(2^n + 4^n)$$

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Multiplikativni inverz od $(1-x)^n$ v $K[[x]]$.

... $(1-x)^{-n} = \sum_{i=0} \binom{i+n-1}{n-1} x^i$

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$$a_{n+1} = -2a_n - 4b_n \qquad n \geq 0 \quad a_0 = 1$$
$$b_{n+1} = 4a_n + 6b_n \qquad b_0 = 0$$

$$x^{n+1} a_{n+1} = -2x^{n+1} a_n - 4x^{n+1} b_n$$

$$x^{n+1} b_{n+1} = x^{n+1} 4a_n + x^{n+1} 6b_n$$

$$\sum_{n=0}^{\infty} a_{n+1} x^{n+1} = -2 \sum_n a_n x^{n+1} - 4 \sum_n b_n x^{n+1}$$

$$\sum_n b_{n+1} x^{n+1} = 4 \sum_n a_n x^{n+1} + 6 \sum b_n x^{n+1}$$

$$A(x) = \sum_n a_n x^n \qquad B(x) = \sum_n b_n x^n$$

$$A(x) - \overset{1}{a_0} = -2A(x)x - 4B(x)x$$
$$B(x) - \overset{0}{b_0} = 4A(x)x + 6B(x)x$$

$$A(x) + 2A(x)x = -4B(x)x + 1$$
$$A(x)(1+2x) = -4B(x)x + 1$$
$$A(x) = \frac{-4B(x)x + 1}{1+2x}$$

$$B(x) = 4\left(\frac{-4B(x)x+1}{1+2x}\right)x + 6B(x)x$$

$$B(x) = \frac{-16B(x)x^2+4x}{1+2x} + 6B(x)x$$

$$B(x)(1+2x) = -16B(x)x^2 + 4x + 6B(x)x + 12B(x)x^2$$

$$B(x) + 2xB(x) + 4B(x)x^2 - 6B(x)x = 4x$$

$$B(x)\left(1-4x+4x^2\right) = -4x$$

$$B(x) = \frac{4x}{(1-2x)^2}$$

$$A(x) = \frac{-4\left(\frac{4x}{(1-2x)^2}\right)x+1}{1+2x} = \frac{\frac{-16x^2}{(1-2x)^2}+1}{1+2x} = \frac{-16x^2+1-4x+4x^2}{(1+2x)(1-2x)^2} =$$

$$= \frac{12x^2-4x+1}{(1-2x)^2(1+2x)} = \frac{\cancel{(1+2x)}(1-6x)}{(1-2x)^2\cancel{(1+2x)}} = \frac{1-6x}{(1-2x)^2}$$

$$A(x) = \frac{1-6x}{(1-2x)^2} = (1-6x)\sum_{i=0}^{\infty}\binom{i+1}{1}(2x)^i = (1-6x)\sum_{i=0}^{\infty}(i+1)(2x)^i =$$

$$= \sum_{i=0}^{\infty}(i+1)(2x)^i - 3\sum_{i=0}^{\infty}(i+1)(2x)^{i+1} =$$

$$= \sum_{i=0}^{\infty}(i+1)(2x)^i - 3\sum_{i=1}^{\infty} i(2x)^i = \sum_{i=0}^{\infty}(i+1)(2x)^i - 3\left(\sum_{i=0}^{\infty} i(2x)^i - 0\right) =$$

$$= \sum_{i=0}^{\infty}(2x)^i(i+1-3i) = \sum_{i=0}^{\infty}(1-2i)2x^i \qquad a_i = 2^i(1-2i)$$