IRZVFRI 2025-01-08

1. $L_e = \{ \langle M \rangle \mid L(M) = \emptyset \}$ ni polodločljiv

empty

dokaz: poiščimo prevedbo $\overline{L_u} \to L_e$.

prevedba je izračunljiva fja $f: \Sigma^* \to \Sigma^*$, da velja

① $\forall z \in \overline{L_u} \Rightarrow f(z) \in L_e \sim \forall \langle M,w\rangle \in \overline{L_u} \Rightarrow \langle M' \rangle \in L_e$

② $\forall z \notin \overline{L_u} \Rightarrow f(z) \notin L_e \sim \forall \langle M,w\rangle \notin \overline{L_u} \Rightarrow \langle M' \rangle \notin L_e$

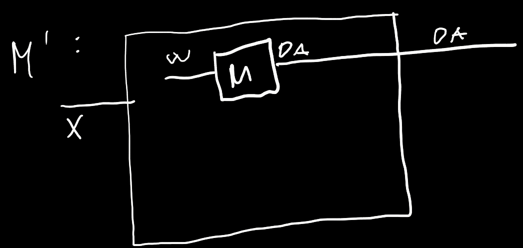
v našem specifičnem primeru $\langle M,w\rangle \mapsto \langle M' \rangle$

$\overline{L_u} = \{ \langle M,w \rangle ; w \notin M \}$

① $\sim \forall w, M \ni: w \notin L(M) \Rightarrow L(M') = \emptyset$

② $\sim \forall w, M \ni: w \in L(M) \Rightarrow L(M') \neq \emptyset$

$f: \langle w, M \rangle \mapsto \langle M' \rangle$



M' ignorira vhod in požene M na w. Če M reče DA, M' reče DA.

f zadošča pogojem prevedbe

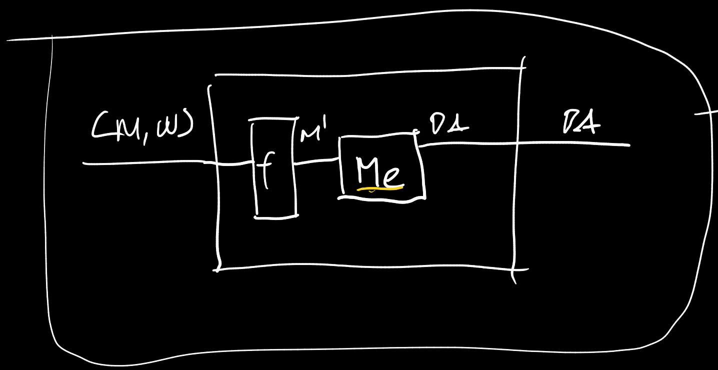
$$L(M') = \begin{cases} \emptyset ; & w \notin L(M) \\ \Sigma^* ; & w \in L(M) \end{cases}$$

če bi bil L_e polodločljiv, bi zaradi obstoja prevedbe na $\overline{L_u}$ le-ta polodločljiv. ✗.

→ $\exists$ stroj, čigar jezik je L_e

→ z njim in prevedbo lahko izdelamo stroj zanj

$$\exists M_e : L(M_e) = L_e \Rightarrow \exists M_{\bar u} : L(M_{\bar u}) = \overline{L_u}$$



→ to bi bil stroj za $\overline{L_u}$. Dokaz:

$$\langle M, w\rangle \in \overline{L_u} \Leftrightarrow w \notin L(M)$$
$$\Updownarrow$$
$$L(\overbrace{f(\langle M, w\rangle)}^{M'}) = \emptyset$$
$$\Updownarrow$$
M_e reče DA
$$\Updownarrow$$
$M_{\bar u}$ reče da

$$\langle M, w\rangle \notin \overline{L_u} \Leftrightarrow w \in L(M)$$
$$\Updownarrow$$
$$L(\overbrace{f(\langle M, w\rangle)}^{M'}) \neq \emptyset$$
$$\Updownarrow$$
M_e ne reče DA
$$\Updownarrow$$
$M_{\bar u}$ ne reče da

torej

$\langle M, w\rangle \in \overline{L_u} \Leftrightarrow M_{\bar u}$ reče DA

$\wedge$ $\langle M, w\rangle \notin \overline{L_u} \Leftrightarrow M_{\bar u}$ ne reče DA

$$\Downarrow$$

$$L(M_{\bar u}) = \overline{L_u}.$$

$\exists M_e \ni : L(M_e) = L_e \Rightarrow \underline{\exists M_{\bar u} \ni : L(M_{\bar u}) = \overline{L_u}}$ ni res

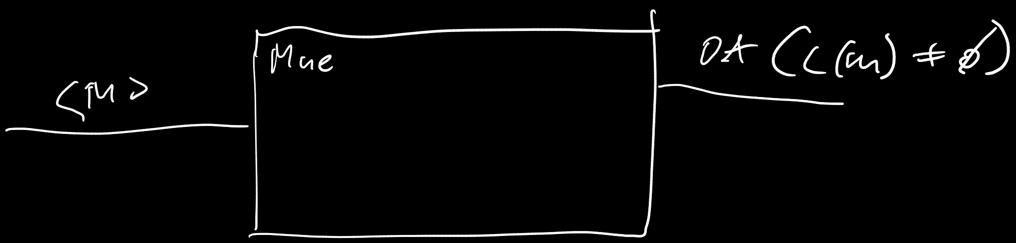
S

$\underbrace{\nexists M_{\bar u} \ni : L(M_{\bar u}) = \overline{L_u}}_{\text{res}} \Rightarrow \underbrace{\nexists M_e \ni : L(M_e) = L_e}_{\text{res}} \Rightarrow L_e$ ni polodločljiv f je očitno izračunljiva

not empty

(2) $L_{ne} = \{\langle M \rangle ; L(M) \neq \emptyset\}$ a je polodločljiv?

(odločljiv gotovo ni, ker $L_e = \overline{L_{ne}}$ ni odločljiv)



generiramo besede ... ali M sprejme w?

w	Ali M sprejme w?
ε	NE
0	NE
1	NE
00	SE zacikla
01	NE
10	NE
⋮	⋮
xx	DA

(w, k)
↳ št. korakov

...

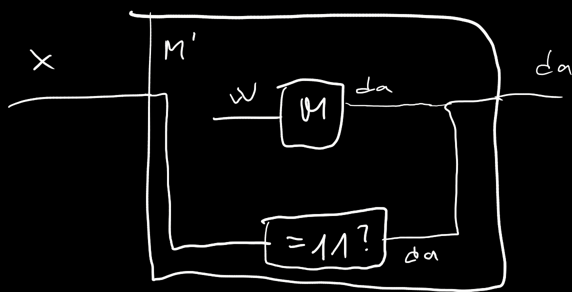
glej preštevanje vaje
(čibej)

3. $L_{11} = \{\langle M\rangle ; L(M) = \{11\}\}$ je nepolodločljiv.

poiščemo prevedbo $\overline{L_u} \to L_{11}$

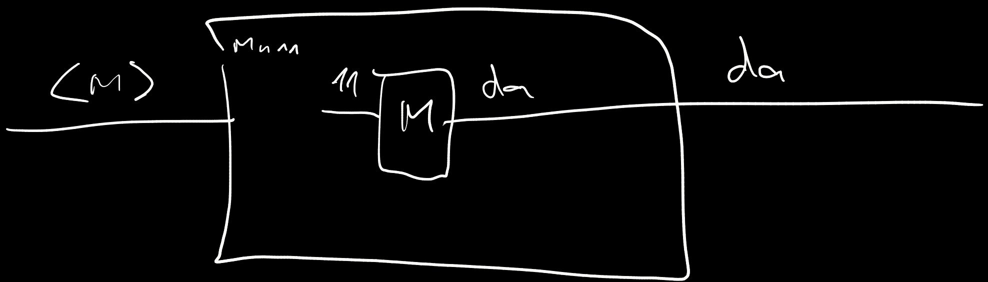
$f : \langle M, w\rangle \mapsto \langle M'\rangle$

1. $\langle M,w\rangle \in \overline{L_u} \to M' \in L_{11} \sim w \notin L(M) \Rightarrow L(M') = \{11\}$
2. $\langle M,w\rangle \notin \overline{L_u} \to M' \notin L_{11} \sim w \in L(M) \Rightarrow L(M') \neq \{11\}$



$$L(M') = \begin{cases} \{11\} & w \notin L(M) \\ \Sigma^* & w \in L(M) \end{cases}$$

4. $L_{N11} = \{\langle M\rangle ; 11 \in L(M)\}$ polodločljiv,
ni pa odločljiv.



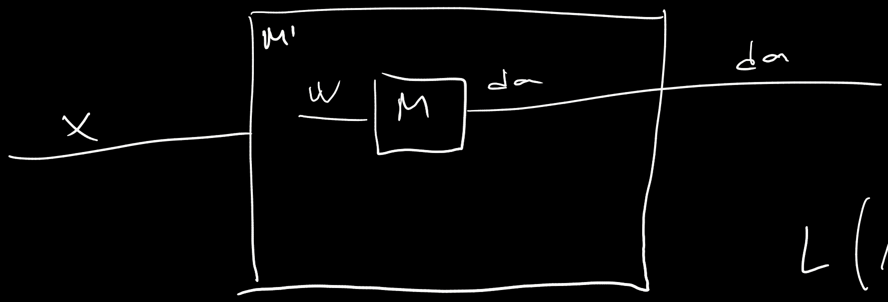
neodločljivost:

$L_u \to L_{N11}$

$f : \langle M, w\rangle \mapsto M'$

(1) $\langle M, w\rangle \in L_u \Rightarrow M' \in L_{uni} \wedge w \in L(M) \Rightarrow M \in L(M')$

(2) $\langle M, w\rangle \notin L_u \Rightarrow M' \notin L_{uni} \wedge w \notin L(M) \Rightarrow M \notin L(M')$



$$L(M') = \begin{cases} \Sigma^* & ; w \in L(M) \\ \emptyset & ; w \notin L(M) \end{cases}$$

ugoboje!!

ne ugoboje !!

(5.) $L_{reg} = \{\langle M\rangle ;\ L(M) \text{ je regularen}\}$

$\overline{L_u} \longrightarrow L_{reg}$

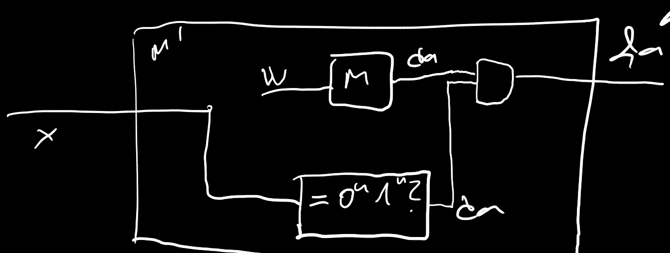
$f: \langle M, w\rangle \rightarrow \langle M'\rangle$

(1) $\langle M, w\rangle \in \overline{L_u} \Rightarrow \langle M'\rangle \in L_{reg}$

(2) $\langle M, w\rangle \notin \overline{L_u} \Rightarrow \langle M'\rangle \notin L_{reg}$

(1) $w \notin L(M) \Rightarrow M'$ je reg

$w \in L(M) \Rightarrow M'$ ni reg



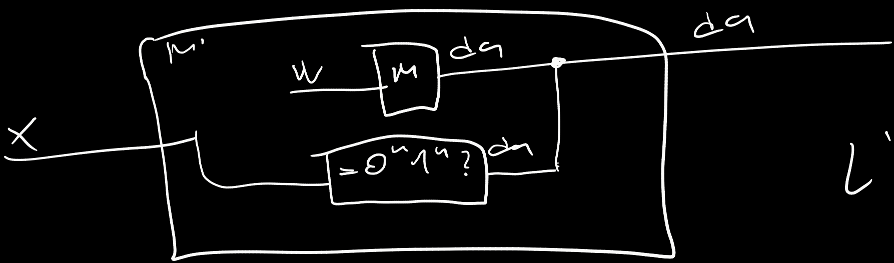
$$L(M') = \begin{cases} \emptyset & ; w \notin L(M) \\ \{0^n1^n ; n \in \mathbb{N}\} & ; w \in L(M) \end{cases}$$

nereg

(6.) $L_{nreg} = \{ \langle M \rangle ;$ $L(M)$ ni regularen $\}$ ni polod.

(1) $w \notin L(M) \Rightarrow L(M')$ ni reg ✓

(2) $w \in L(M) \Rightarrow L(M')$ je reg ✓



$$L'(M') = \begin{cases} \{0^n1^n ; n \in \mathbb{N}\} & w \notin L(M) \\ \Sigma^* & w \in L(M) \end{cases}$$

✓

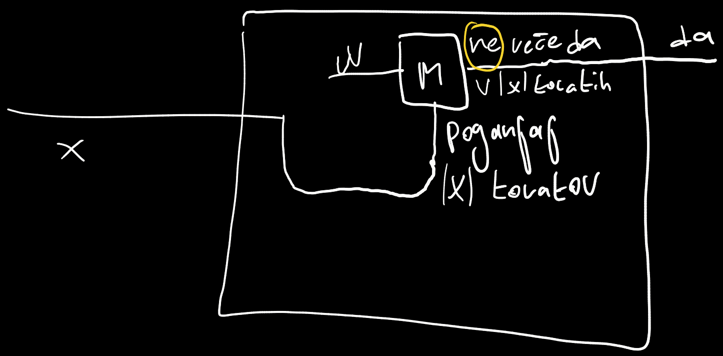
(7.)

$L_\infty = \{ \langle M \rangle ; |L(M)| = |\mathbb{N}| \}$ ni polodločljiv

$\overline{L_u} \longrightarrow L_\infty$

(1) $w \notin L(M) \rightarrow |L(M')| = |\mathbb{N}|$

(2) $w \in L(M) \rightarrow |L(M')| < |\mathbb{N}|$



M nikoli ne vrne da

$$L(M') = \begin{cases} \Sigma^* & w \notin L(M) \\ \underbrace{\{w \in \Sigma^* ; |w| < k\}}_{\text{končna}} & w \in L(M) \end{cases}$$

po k korakih M vrne DA