ANA3VFMF 2024-11-25

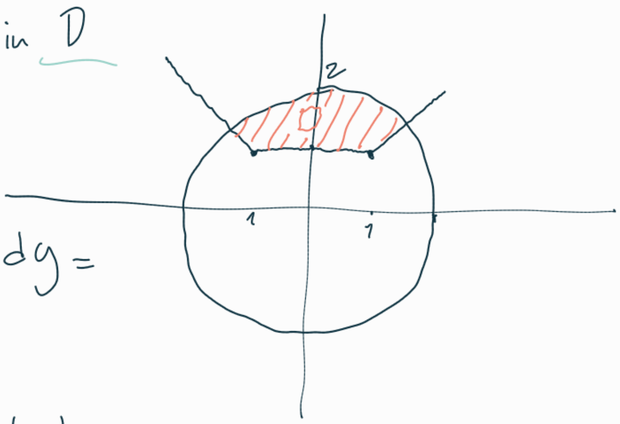
let $D = \{(x,y)\in\mathbb{R}^2;\ x^2+y^2 \le 4 \ \wedge\ y > \max\{|x|, 1\}\}$

lik s točkovno gostoto $\rho(x,y) = (x^2+y^2)^{-3}$.

Izračunaj masno središče D

simetrija ρ in D

$$\bar{x} = 0$$



$$\bar{y} = \frac{1}{m(D)} \iint_D \rho(x,y)\, y \, dx\,dy =$$

$$= \frac{1}{m(D)} \iint_D (x^2+y^2)^{-3}\, y \, dx\,dy = \text{polarne}$$

$x = r\cos\varphi \quad y = r\sin\varphi$

$x^2+y^2 = r^2 \quad \det J = r$

$r\sin\varphi = 1$

$r = \frac{1}{\sin\varphi}$

$$= \frac{1}{m(D)} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{\frac{1}{\sin\varphi}}^{2} r^{-6}\, r\sin\varphi \cdot r \, dr\,d\varphi =$$

$$= \frac{1}{m(D)} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{\frac{1}{\sin\varphi}}^{2} r^2 r^{-6} \sin\varphi \, dr\,d\varphi =$$

$$= \frac{1}{m(D)} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{\frac{1}{\sin\varphi}}^{2} r^{-4} \sin\varphi \, dr\,d\varphi = \frac{1}{m(D)} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} -\frac{1}{3} r^{-3} \sin\varphi \Big|_{\frac{1}{\sin\varphi}}^{2} d\varphi =$$

$$= \frac{1}{m(D)} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} -\frac{1}{3}\cdot\frac{1}{8}\cdot\sin\varphi + \frac{1}{3}\cdot\sin^3\varphi\cdot\sin\varphi \, d\varphi =$$

$$= \frac{1}{m(D)} \left(\cos\varphi \cdot \frac{1}{24} \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + \frac{1}{3} \boxed{\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^4\varphi \, d\varphi} \right) = \frac{1}{m(D)} \left(\frac{\sqrt{2}}{24} + \frac{1}{3} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^3\varphi \sin\varphi \, d\varphi \right) =$$

$u = \sin^3\varphi \quad du = \sin^2\varphi \cos\varphi\, d\varphi$

$dv = \sin\varphi\, d\varphi \quad v = -\cos\varphi$

$$= \frac{1}{m(D)} \left(\frac{\sqrt{2}}{24} + \frac{1}{3} \left(-\sin^3\varphi\cos\varphi + 3 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\varphi \underbrace{\cos^2\varphi}_{1-\sin^2\varphi} \, d\varphi \right) \right) =$$

$$= \frac{1}{m(D)} \left(\frac{\sqrt{2}}{24} + \frac{1}{3} \left(\overbrace{-\left(\frac{\sqrt{2}}{2}\right)^3 \left(-\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)^3 \frac{\sqrt{2}}{2}}^{1/2} + 3 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\varphi \, d\varphi - 3 \boxed{\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^4\varphi \, d\varphi} \right) \right) =$$

rekurzivni obrazec

$$= \frac{1}{m(D)} \left(\frac{\sqrt{2}}{24} + \frac{1}{3} \left(\frac{1}{2} + 3 \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\varphi\, d\varphi - 3 \left(\frac{1}{8} + \frac{3}{8} \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sin^2\varphi \, d\varphi \right) \right) \right) = \ldots \text{ D.N.}$$

N

1. kol. 23/24: 2. naloga (D.N.)

vztrajnostni moment itd.

N

Dokaži, da nove spremenljivke $u = y + xyz$ podajajo injektivno preslikavo
$v = x + xyz$
$w = x + y$

na območju $\Omega = \{(x,y,z) \in \mathbb{R}^3 ;\ xy > 0\}$ in določi $F(\Omega)$.

- poiščimo inverz. izrazimo x, y, z z u, v, w

$$y = w - x \qquad x = \frac{v}{1+yz}$$
$$x = w - y \qquad y = \frac{u}{1+xz}$$

$$x = w - y = w - \frac{u}{1+xz}$$
$$x(1+xz) = w(1+xz) - u$$
$$x + x^2 z = w + xzw - u$$
$$x + x^2 z - xzw = w - u$$
$$x^2 z - xzw = w - u - x$$
$$z(x^2 - xw) = w - u - x$$
$$z = \frac{w-u-x}{x^2 - xw}$$

izrazi x brez y, z:

$$v = x + \cancel{x} \frac{u}{1+\cancel{x}\left(\frac{w-u-x}{x^{\cancel{2}} - \cancel{x}w}\right)} \cdot \frac{w-u-x}{x^{\cancel{2}} - w\cancel{x}} =$$

$$= x + \frac{u}{1+\frac{w-u-x}{\cancel{x-w}}} \cdot \frac{w-u-x}{\cancel{x-w}} = x + \frac{u(w-u-x)}{\cancel{x} \cancel{-w} \cancel{+w} -u \cancel{-x}} =$$

$$= x - w + u + x \Rightarrow 2x = v + w - u$$

$$x = \frac{v+w-u}{2}, \quad y = \frac{u+w-v}{2}$$

$$z = \frac{w - u - \frac{v+w-u}{2}}{\frac{(v+w-u)^2}{4} - \frac{vw + w^2 - uw}{2}} = \text{poenostavi} \ldots$$