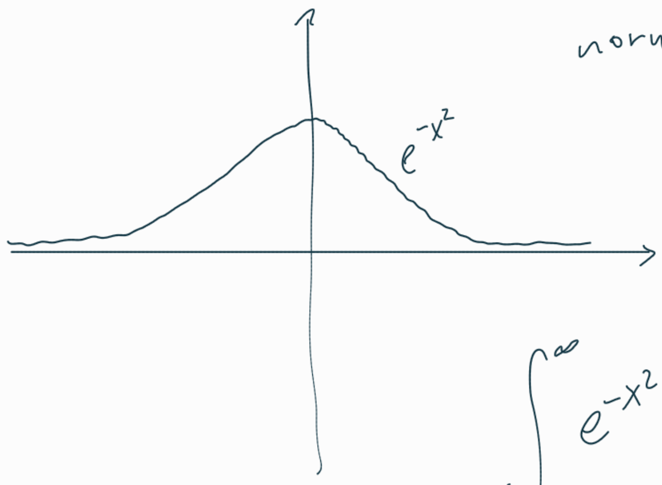
ANA3 VFMF 2024-11-19

normalna porazdelitev



$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \dots \text{ gamma} \dots$$

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right)\left(\int_{-\infty}^{\infty} e^{-y^2}\,dy\right) =$$

$$\overset{\text{fubini}}{=} \iint_{\mathbb{R}^2} e^{-x^2-y^2}\,dx\,dy = \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\,dx\,dy \underset{\text{polar}}{=} \int_0^{2\pi}\int_0^{\infty} e^{-R^2}\cdot R\,dR\,d\varphi =$$

pozitivna f-ja

$$= 2\pi \int_0^{\infty} e^{-R^2}\cdot R\,dR = \cancel{2}\pi \int_0^{\infty} e^{-t}\,\cancel{\sqrt{t}}\,\frac{1}{\cancel{2}\cancel{\sqrt{t}}}\,dt = \pi\int_0^{\infty} e^{-t}\,dt =$$

$t = R^2$

$R = \sqrt{t}$

$dR = \frac{1}{2\sqrt{t}}\,dt$

$$= \pi\left(-e^{-t}\right)\Big|_0^{\infty} = \pi$$

$$\Downarrow$$

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

N

Za katere $p \in \mathbb{R}$ konvergira $\iiint_{\mathbb{R}^3} e^{-(x^2+y^2+z^2)^p}\,dx\,dy\,dz$?

Kadar konvergira, ga izračunaj.

$$\iiint_{\mathbb{R}^3} e^{-(x^2+y^2+z^2)^p}\,dx\,dy\,dz \overset{\text{sferične}}{=} \iiint_{[0,\infty)_r,\,[0,2\pi)_\varphi,\,(-\frac{\pi}{2},\frac{\pi}{2})_\theta} e^{-r^{2p}}\cdot r^2\cos\theta\,d\varphi\,d\theta\,dr \overset{\text{fubini}}{=}$$

pozitivna f-ja

$$= 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{\infty} e^{-r^{2p}}\,r^2\cos\theta\,d\theta\,dr = 2\pi\int_0^{\infty} e^{-r^{2p}}\,r^2 \sin\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\,dr =$$

za $p \le 0$ $\to 1$ ne konvergira, torej gledamo le še $p > 0$

$$= 4\pi\int_0^{\infty} e^{-r^{2p}}\,r^2\,dr = 4\pi\int_0^{\infty} e^{-u}\,u^{1/p}\cdot\frac{1}{2p}\cdot u^{\frac{1}{2p}-1}\,du =$$

$u = r^{2p}$ $\quad r = u^{\frac{1}{2p}}$

$dr = \frac{1}{2p}\,u^{\frac{1}{2p}-1}\,du$

$$= 2\pi\frac{1}{p}\int_0^{\infty} e^{-u}\,u^{\frac{1}{p}+\frac{1}{2p}-1}\,du =$$

$$= \frac{2\pi}{p}\,\Gamma\left(\frac{3}{2p}\right)$$

let $p>0$. izračunaj $\iiint_{\mathbb{R}^3} \sin(z)\, e^{-(x^2+y^2+z^2)^p}\,dx\,dy\,dz =$

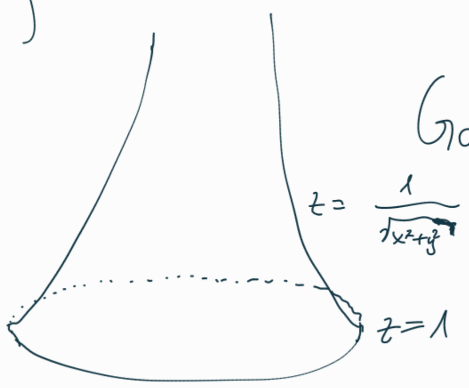
$= \ldots = 0$

N

let $T = \{(x,y,z)\in\mathbb{R}^3;\ z^2(x^2+y^2) < 1,\ z \geq 1\}$

$$z \leq \frac{1}{\sqrt{x^2+y^2}}$$

izračunaj $Vol(T)$



Gabrielov rog

$$V = \iiint_T 1\,dx\,dy\,dz \overset{\text{cilindrične}}{=} \iiint_\Omega r\,dr\,d\varphi\,dz =$$

$$\Omega = (0,1)_r \times [0,2\pi]_\varphi \times \left[1,\tfrac{1}{r}\right]_z$$

$$= 2\pi \int_0^1 zr \int_1^{\frac{1}{r}} dr = 2\pi \int_0^1 1-r\,dr = 2\pi\; r-\frac{r^2}{2}\Big|_0^1 = \pi$$

b.) $\rho(x,y,z) = \frac{1}{z}$ je gostota T.

izračunaj masno središče.

$\bar{x} = \bar{y} = 0$ (simetrija)

$$\bar{z} = ? = \frac{1}{m(T)} \iiint_T \rho(x,y,z)\, z\,dx\,dy\,dz =$$

$$= \frac{1}{m(T)} \iiint_T \cancel{\frac{1}{z}}\,\cancel{z}^{\,1}\,dx\,dy\,dz = \frac{1}{m(T)}\, Vol(T) = \frac{\pi}{m(T)}$$

$$m(T) = \iiint_T \rho(x,y,z)\, dx\,dy\,dz = \int_0^1 dr \int_0^{2\pi} d\varphi \int_1^{\frac{1}{r}} \frac{r}{z}\, dz$$

$$= 2\pi \int_0^1 dr \left(r \cdot \ln(z) \Big|_1^{\frac{1}{r}} \right) = 2\pi \int_0^1 r \ln\left(\frac{1}{r}\right) dr =$$

$$= -2\pi \int_0^1 r \ln r \, dr = -\cancel{2}\pi \left(\ln r \cdot \frac{r^2}{\cancel{2}} \Big|_0^1 - \int_0^1 \frac{r^2}{\cancel{2}} \cdot \frac{1}{\cancel{r}}\, dr \right) =$$

$\ln r = u$

$\frac{1}{r} = \frac{du}{dr}$

$r\,dr = dv$

$\frac{1}{r} r^2 = v$

$$= -\pi \left(\ln r \, r^2 \Big|_0^1 - \frac{r^2}{2} \Big|_0^1 \right) =$$

$$= -\pi \left(\frac{1}{2} - \frac{0}{2} \right) = \frac{\pi}{2} \quad \checkmark$$

$$\bar{z} = \frac{\pi}{\pi/2} = 2$$

center mase: $(0,0,2)$

c.) izračunaj vztrajnostni moment za vrtenje T okoli osi z.

$$J_z = \iiint_T (x^2+y^2)\,\rho(x,y,z)\, dx\,dy\,dz$$

P. N.