ANA3UFMF2024-11-04

izpiti:

pet, 31.1.
tor, 11.2. ob 10h
čet, 21.8.

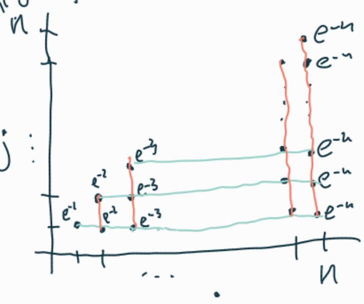
N

Izračunaj $\sum_{i=1}^{n} i\cdot e^{-i}$ za $n\in\mathbb{N}$.

1. način: $\left(\frac{\partial}{\partial x}\sum_{i=1}^{n} e^{-xi}\right)$ in vstavimo $x=1$ v ta odvod

2. način: zamenjaj iz $\sum_{j=1}^{i} 1$:

oklepaji zaradi distrib. niso potrebni!



$$\sum_{i=1}^{n}\left(\sum_{j=1}^{i} 1\right)e^{-i}$$

zamenjaj vrstni red seštevanja

oklep. -||-

$$\sum_{j=1}^{n}\left(\sum_{i=j}^{n} 1\right)e^{-i} = \sum_{j=1}^{n}\sum_{i=1}^{n} 1\cdot e^{-i} = \sum_{j=1}^{n}\underbrace{\sum_{i=1}^{n} e^{-i}}_{\text{geom. vrsta}} =$$

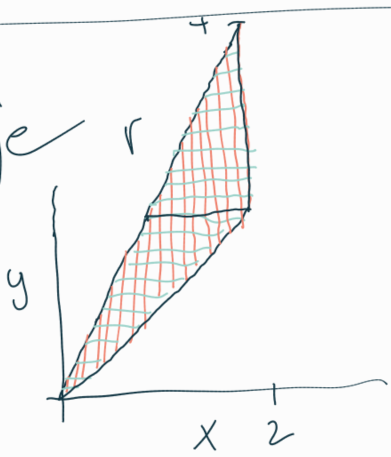
$$= \sum_{j=1}^{n}\left(e^{-j}+e^{-(j+1)}+e^{-(j+2)}+\dots+e^{-n}\right) = \sum_{j=1}^{n} e^{-j}\left(1+e^{-1}+e^{-2}+\dots+e^{-(n-j)}\right) =$$

$$= \sum_{j=1}^{n} e^{-j}\cdot\frac{1-e^{-n+j-1}}{1-e^{-1}} = \frac{1}{1-e^{-1}}\sum_{j=1}^{n} e^{-j}\cdot\left(1-e^{-n+j-1}\right) =$$

$$= \frac{1}{1-e^{-1}}\left(\left(\sum_{j=1}^{n} e^{-j}\right) - \sum_{j=1}^{n} e^{-n-1}\right) = \frac{1}{1-e^{-1}}\left(e^{-1}\cdot\frac{1-e^{-n-1}}{1-e^{-1}} - n\cdot e^{-n-1}\right)$$

N

Zamenjaj vrstni red integracije

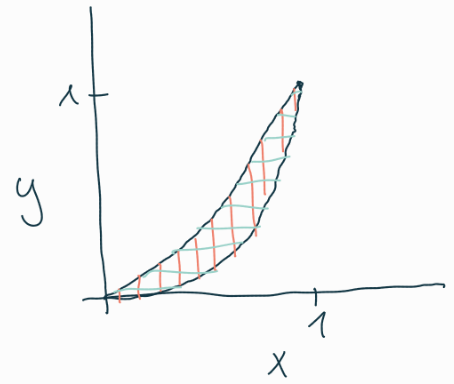


a) $\int_0^2\left(\int_x^{2x} f(x,y)\,dy\right)dx$

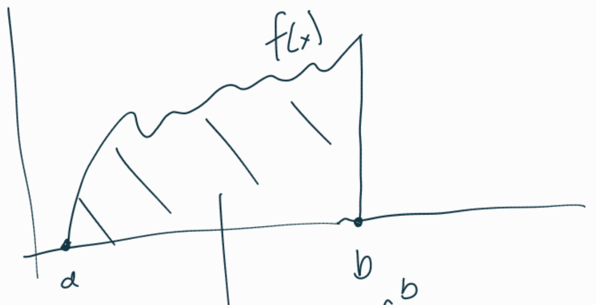
b) $\int_0^1\left(\int_{x^3}^{x^2} f(x,y)\,dy\right)dx$

a) $\int_0^2\left(\int_{\frac{y}{2}}^{y} f(x,y)\,dx\right)dy + \int_2^4\left(\int_{\frac{y}{2}}^{2} f(x,y)\,dx\right)dy$

b.) $\displaystyle\int_0^1 \left(\int_{\sqrt{y}}^{\sqrt[3]{y}} f(x,y)\,dx \right) dy$



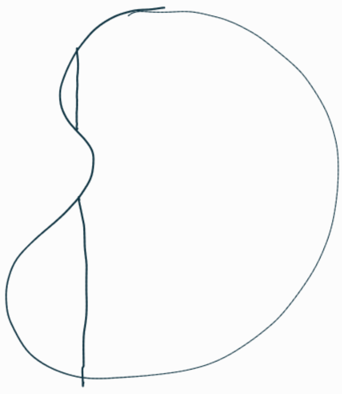
[Dvojni integral]



↳ pl: $\displaystyle\int_a^b f(x)\,dx$

↳ volumen:

$$\iint_D f(x,y)\,dx\,dy$$



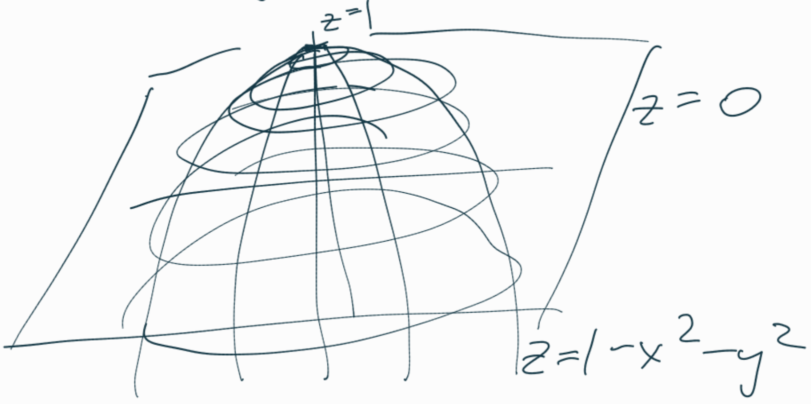
Fubini: $\displaystyle\int_a^b \left(\int_{c(x)}^{d(x)} f(x,y)\,dy \right) dx$

META: Dodatni termin za nadomeščanja:

pon; 15h–17h

N

Izračunaj volumen telesa, omejenega s ploskvama $z = 1 - x^2 - y^2$ & $z = 0$



$$V = \iint_D (1 - x^2 - y^2)\,dx\,dy$$

kaj je D?

za katere (x,y) $\exists z$ $\ni$: $(x,y,z) \in$ telo

enotski krog

$1 \geq x^2 + y^2 \Leftarrow\ \Leftrightarrow\ 0 \leq 1 - x^2 - y^2$

$$V = \int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \left(1-x^2-y^2\right)dx\,dy =$$

$$= \int_{-1}^{1} \left(x - xy^2 - \frac{x^3}{3}\right)\Bigg|_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy =$$

$$= \int_{-1}^{1} 2\left(\sqrt{1-y^2} - \sqrt{1-y^2}\cdot y^2 - \frac{(1-y^2)^{\frac{3}{2}}}{3}\right) dy =$$

$$= 2\int_{-1}^{1} \sqrt{1-y^2}\left(1-y^2-\frac{1-y^2}{3}\right) dy =$$

$$= 2\int_{-1}^{1} \sqrt{1-y^2}\cdot\frac{3}{2}\, dy = \boxed{\frac{8}{3}}\int_{0}^{1} \left(1-y^2\right)^{3/2} dy = \frac{8}{3}\int_0^1 \frac{1}{2} u^{-\frac{1}{2}}\left(1-u\right)^{\frac{3}{2}} du =$$

$$u = y^2 \qquad y = \sqrt{u}$$

$$dy = \frac{1}{2\sqrt{u}}\, du$$

$$p-1 = -\frac{1}{2} \qquad p = \frac{1}{2}$$

$$q-1 = 3/2 \qquad q = 5/2$$

$$= \frac{4}{3}\cdot B\left(\frac{1}{2}, \frac{5}{2}\right) = \frac{4}{3}\,\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{5}{2}\right)}{\Gamma(3)} = \frac{4}{3}\cdot\frac{\pi\cdot 3}{8} = \frac{\pi}{2}$$