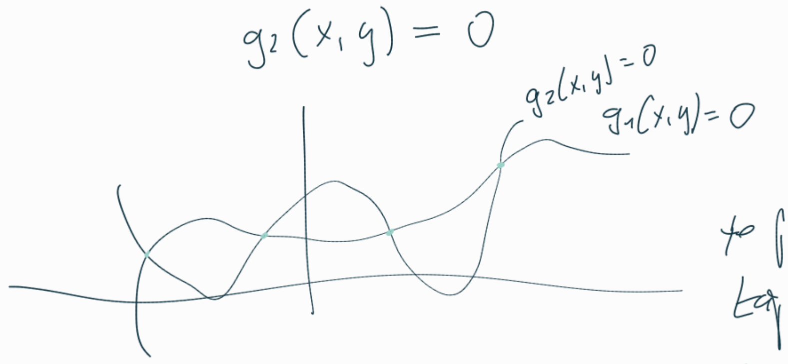
VEZANI EKSTREM, toda z več pogoji

Iščemo ekstreme $f(x,y)$

$g_1(x,y)=0$ in

$g_2(x,y)=0$



to je lahko. Kaj pa več spremenljivk?

Iščemo ekstreme f-je n spremenljivk $f(x_1,\dots,x_n)$ pri k pogojih:

$$g_1(x_1,\dots,x_n)=0$$
$$\dots$$
$$g_k(x_1,\dots,x_n)=0$$

$$F(x_1,\dots,x_n,\lambda_1,\dots,\lambda_k)=f(x_1,\dots,x_n)-\lambda_1 g_1(\dots)-\dots-\lambda_k g_k(\dots)$$

N

Poišči vezane VVJ

$$x^2+y^2=z^2$$
$$x-2z=3$$

A lahko problem iskanja ekstremov predelamo na iskanje vezanega ekstrema?

$\varphi(x,y,z)=z$ pri pogoju $z-\underbrace{f(x,y)}=0$

↓

iščemo ekstrem tega.

great, samo navedili smo iskanje ekstrema.

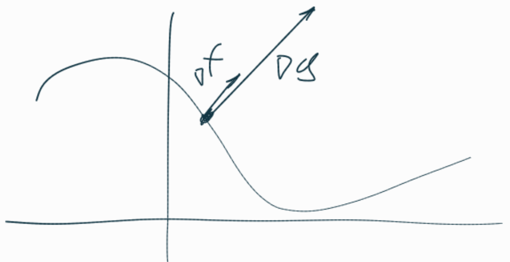
Aplikacija λ:

$f(x,y)$ pri pogoju $g(x,y)=0$

$\Downarrow$

$\nabla f \parallel \nabla g$

$\hookrightarrow \nabla f=\lambda\nabla g$



KRIVULJE V $\mathbb{R}^{\{2,3\}}$

· eksplicitno, implicitno, parametrično

↓

če je r vektor parametrizacijskih funkcij, je $K = \operatorname{im}(r)$ tir krivulje

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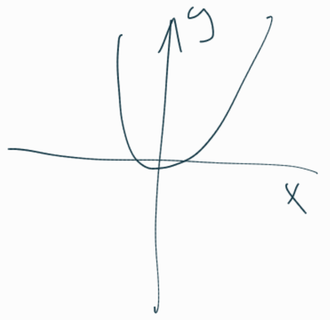
Primer

$K = \{(x,y) \in \mathbb{R}^2;\ y = x^2\}$



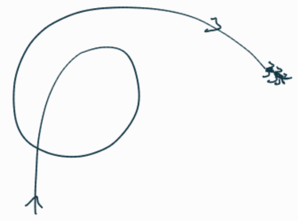
iščemo $\vec{r} = (x(t), y(t))$

$x(t) = t$
$y(t) = t^2 \quad t \in (-\infty, \infty)$

ali pa $x(t) = e^t \sin t$
$y(t) = e^{2t} \sin^2 t \quad t \in [0, \infty)$

f&q:

„mravlja ne pride v nobeno protislovje če seka svojo pot"
—asistent



„obstaja ant death circle"

—barbara.

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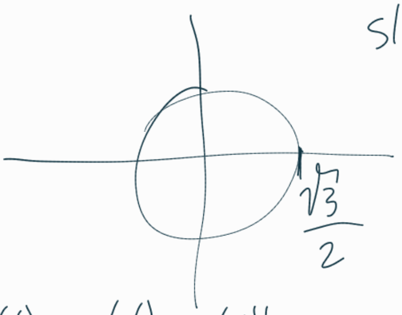
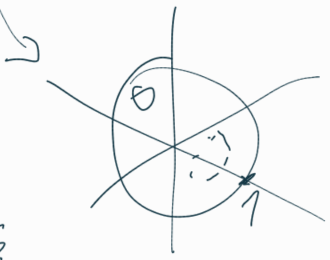
$K = \{(x,y,z) \in \mathbb{R}^3;\ \underbrace{x^2+y^2+z^2 = 1} \text{ in } z = \frac{1}{2}\}$

$x^2 + y^2 = \boxed{3/4}$

slice na $z = 1/2$





$r = \frac{\sqrt{3}}{2}$

$s = (0,0,0)$

$\vec{r} = (x(t), y(t), z(t))$

$z(t) = 1/2$
$x(t) = (\cos t)\left(\frac{\sqrt{3}}{2}\right)$
$y(t) = (\sin t)\left(\frac{\sqrt{3}}{2}\right)$

$t \in [0, 2\pi)$

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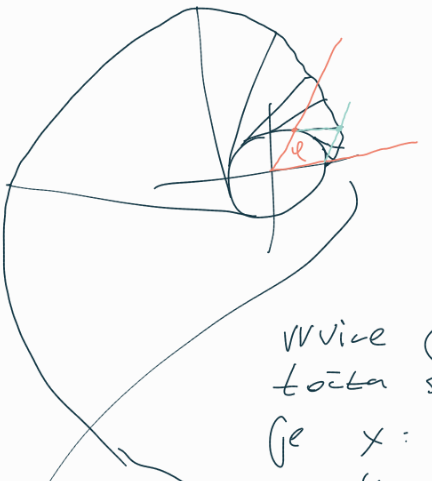
Parametriziraj krivuljo, ki jo opiše točka kroga radija R, ko se ta brez zdrsavanja kotali po premici

Cikloida eksplicitno?

z izrekom o implicitni fji ne moremo trditi, da obstaja cikloida kot eksplicitna fja v $0, 2\pi, 4\pi$.

Parametriziraj krivuljo, ki opiše konec niti, ko se nit odvija s kroga radija R.



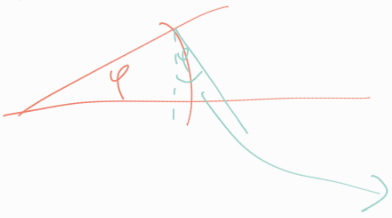
$\vec{r}(t) = (x(t), y(t))$

$x(0) = R$

$y(0) = 0$

kot

vrvice je φ, njena točka stika s krogom je $x: R\cos\varphi$, $y: R\sin\varphi$



$\vec{r}(t) = \left(R\cos t + Rt\sin t,\ R\sin t - Rt\cos t\right)$

$t \in [0, \infty)$

odvod poti in tangentni vektor:

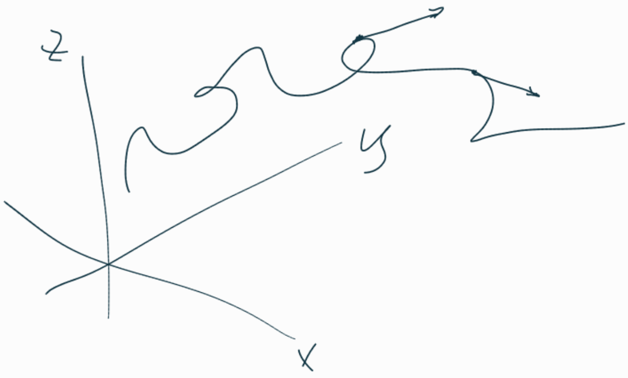
$\vec{r}: I \to \mathbb{R}^3$

$\vec{r}(t) = (x(t), y(t), z(t))$

odvod

$\dot{\vec{r}}(t) = (\dot{x}(t), \dot{y}(t), \dot{z}(t))$

tangentni vektor ob t



Dana je pot

$\vec{r}(t) = \left(\frac{1+t}{t^3},\ \frac{5\sqrt{3}}{3} \cdot \frac{1+t}{t^2}\right)$ $t \in (0, \infty)$

Izračunaj kot, ki ga oklepa tangentni vektor v $t=0$ z x-osjo

kje pa smo v $t=1$?

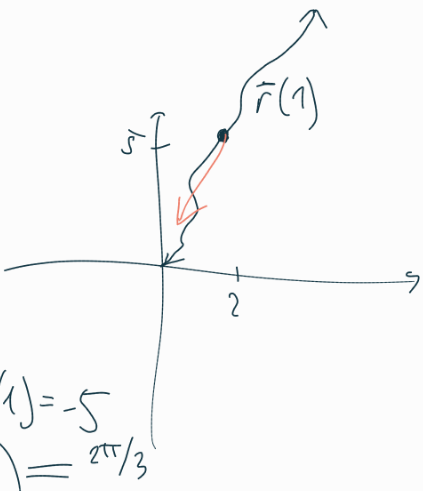
$\dot{x}(t) = \frac{t^3 - (1+t)3t^2}{t^6} = \frac{-t-3}{t^4}$

$\dot{y}(t) = \frac{5\sqrt{3}}{3} \cdot \frac{t^2 - (1+t)2t}{t^4} = \frac{5\sqrt{3}}{3} \cdot \frac{-t-2}{t^3}$

$\dot{\vec{r}}(1) = (-5, -5\sqrt{3})$

kot med vektorja:

$\arccos\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$



$|\vec{i}| = 1$

$|\dot{\vec{r}}(1)| = 10$

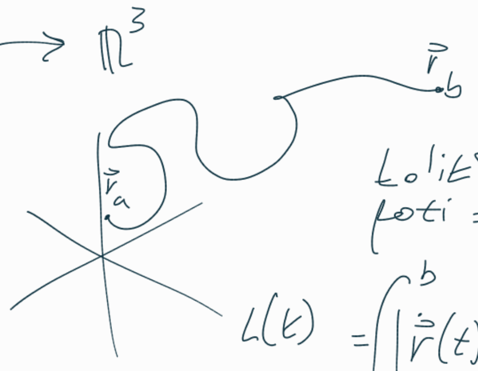
$\vec{i} \cdot \dot{\vec{r}}(1) = -5$

$\arccos\left(-\frac{1}{2}\right) = 2\pi/3$

uporabne naloge:

• dolžina poti: $\vec{r}(t) = (x, y, z)(t)$

$\vec{r}: [a, b] \to \mathbb{R}^3$



Kolikšna je dolžina poti $= L$?

$$L(t) = \int_a^b |\dot{\vec{r}}(t)|\, dt$$

za dolžino želimo injektivno parametrizacijo

↳ oziroma je dovolj že regularna parametrizacija za nesklenjene krivulje

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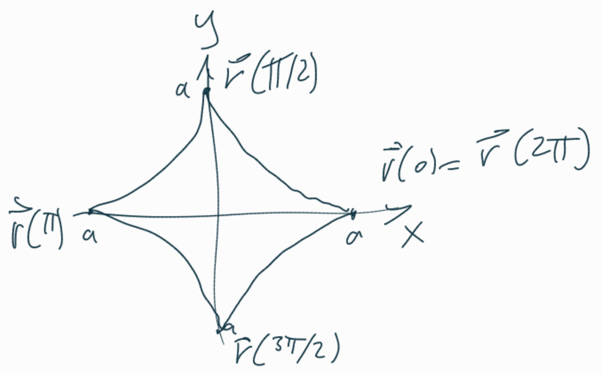
let $K \subset \mathbb{R}^2$ krivulja, parametrizirana z

$\vec{r}(t) = (a\cos^3 t,\ a\sin^3 t)$, $\quad t \in [0, 2\pi)$ $\quad (a > 0)$



Določi dolžino K!

$$\dot{\vec{r}}(t) = (-3a\cos^2 t \sin t,\ 3a\sin^2 t \cos t) =$$

$$= 3a\sin t\cos t(\cos t, \sin t)$$