N

S pomočjo Taylorjeve vrste za eksponentno f/o izračunaj $e$ z napako, manjšo od $10^{-5}$.

$$f(x)=e^x=\sum_{i=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{4!}+\dots$$

želimo $f(1)=e$

$$1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{4!}+\dots$$

$$e=\sum_{i=0}^{\infty}\frac{1}{i!}$$

$\xi\in(0,1)$

$\xi\in(a,x)$

iščemo $R_f(1)=\frac{f^{(n+1)}(\xi)}{(n+1)!}\underbrace{(x-a)^n}_{1}$

$$=\frac{e^{\xi}}{(n+1)!}1=\frac{e^{\xi}}{(n+1)!}<$$ vzamemo največje

↳ $\xi=1,\ e^{\xi}=e$

$$<\frac{e^1}{(n+1)!}=\frac{e}{(n+1)!}<\frac{3}{(n+1)!}$$

$$\frac{3}{(n+1)!}<10^{-5}$$

$$\frac{(n+1)!}{3}>10^5$$

$$(n+1)!>3\cdot10^5$$

(s kalkulatorjem)

$$n\geq 8$$

N

INTEGRALI

uvedjet nova spremenljivka: (odvod kompozituma)

uvedjet 2 per partes (odvod produkta)

$$\int\frac{x^2}{(x-1)^5}\,dx=\int\frac{(u+1)^2}{u^5}\,du=\int\frac{u^2+2u+1}{u^5}\,du=$$

$u=x-1$

$du=1dx$

$dx=1du$

$x=u+1$

$$=\int\frac{u^2}{u^5}\,du+2\int\frac{u}{u^5}\,du+\int\frac{1}{u^5}\,du=$$

$$=\int u^{-3}\,du+2\int u^{-4}\,du+\int u^{-5}\,du=$$

$$=\frac{t^{-2}}{-2}+\frac{2t^{-3}}{-3}+\frac{t^{-4}}{-4}-42\cdot\sqrt[3]{C}$$

$$=\frac{(x-1)^{-2}}{-2}+\frac{2(x-1)^{-3}}{-3}+\frac{(x-1)^{-4}}{-4}-42\cdot\sqrt[3]{C}$$

N

$$\int\frac{1}{\sqrt{x}\,(1+\sqrt[3]{x})}\,dx=\int\frac{6u^5}{u^3(1+u^2)}\,du=\int\frac{6u^2}{1+u^2}\,du=$$

$u=\sqrt[6]{x}$

$x=u^6$

$dx=6u^5\,du$

$$=6\int\left(1-\frac{1}{1+u^2}\right)du=6t-\arctan t-42\sqrt[3]{C}=$$

$$=6\sqrt[6]{x}-\arctan\sqrt[6]{x}-42\sqrt[3]{C}$$

$u^2:u^2+1=1$

ost: $-1$

N

$$\int x\cdot e^{x^2}\,dx = \int e^u \cdot \cancel{x}\,\frac{du}{2\cancel{x}} = \frac{1}{2}\int e^u\,du =$$

$u = x^2$
$du = 2x\,dx$

$$= \frac{1}{2}e^u - 42\sqrt[3]{C} =$$

ovodje #2:

$$= \frac{e^{x^2}}{2} - 42\sqrt[3]{C}$$

Per partes:

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

a) $\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x + C =$
$= e^x(x-1) + C$

b.) $= \int x^3 e^{\frac{x}{2}}\,dx = \int 16u^3 e^u\,du$

$u = \frac{x}{2}$
$du = \frac{1}{2}dx$
$dx = 2du$

$= \dots$

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$$\int e^x\cos x\,dx = e^x\cos x - \int e^x \sin x\,dx = e^x\cos x + e^x\sin x - \int e^x\cos x\,dx$$

$u = e^x$ $\quad dv = \cos x\,dx$
$du = e^x dx$ $\quad v = \sin x$

$u = e^x$ $\quad dv = \sin x\,dx$
$du = e^x dx$ $\quad v = -\cos x$

$$\int e^x\cos x\,dx = e^x\,\frac{\sin x + \cos x}{2} + C$$

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za $r \in \mathbb{R}\setminus\{-1\}$

$$\int x^r \ln x\,dx = x^r \ln x - \int \frac{x^{r+1}}{(r+1)\cancel{x}}\,dx = \dots$$

$u = \ln x$ $\quad dv = x^r$
$du = dx/x$ $\quad v = \frac{x^{r+1}}{r+1}$

---

$$\int x^5\sqrt{x^3+1}\,dx = \int x^3 \cancel{x^2}\sqrt{u}\,\frac{du}{3\cancel{x^2}} =$$

$u = x^3+1$
$du = 3x^2\,dx$
$dx = \frac{du}{3x^2}$

$$= \int x^3\sqrt{u}\,\frac{du}{3} =$$

$$\int x^r = \frac{x^{r+1}}{r+1}$$

$$= \int (u-1)\,u^{1/2}\,\frac{du}{3} =$$

$$= \frac{1}{3}\int (u-1)u^{\frac{1}{2}}\,du = \frac{1}{3}\int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du =$$

$$= \frac{1}{3}\cdot\frac{u^{5/2}}{5/2} - \frac{1}{3}\,\frac{u^{3/2}}{3/2} + C = \frac{2}{3}u^{3/2}\left(\frac{1}{5}u - \frac{1}{3}\right) + C = \frac{2}{3}(x^3+1)\left(\frac{1}{5}(x^3+1) - \frac{1}{3}\right)$$

SNOV: [PARCIALNI ULOMKI]

$$\frac{p(x)}{(x-x_1)^{n_1}\cdots(x-x_k)^{n_k}}$$ če je stopnja števca manjša od stopnje imenovalca, velja

$$= \frac{p_1(x)}{(x-x_1)^{n_1}} + \dots + \frac{p_k(x)}{(x-x_k)^{n_k}}$$

N

$$\int \frac{x-2}{(x+1)(x-1)^2}\,dx$$

$$\frac{x-2}{(x+1)(x-1)^2} = \frac{a}{x+1} + \frac{b}{x-1} + \frac{c}{(x-1)^2} \qquad / \cdot (x+1)(x-1)^2$$

$$x-2 = a(x-1)^2 + b(x+1)(x-1) + c(x+1)$$

$x=1$: $\qquad$ $x=-1$:

$$-1 = 2c \qquad -3 = a(-2)^2 = 4a$$

$$c = \frac{-1}{2} \qquad a = \frac{-3}{4}$$

$x=0$

$$-2 = a - b + c$$

$$-2 = \frac{-1}{2} - \frac{3}{4} - b$$

$$b = \frac{-1}{2} + 2 - \frac{3}{4}$$

$$b = \frac{3}{4}$$

$$\int \frac{x-2}{(x+1)(x-1)^2}\,dx = \int \frac{-3/4}{x+1}\,dx + \int \frac{3/4}{x-1}\,dx + \int \frac{-1/2}{(x-1)^2}\,dx =$$

$$= -\frac{3}{4}\ln|x+1| + \frac{3}{4}\ln|x-1| - \frac{1}{2}(-1)(x-1)^{-1} + C =$$

$$= \frac{3}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2(x-1)} + C$$

N

$$\int \frac{e^{2x}-2}{e^x+1}\,dx = \qquad e^x = t, \quad e^x dx = 1\,dt, \quad dx = \frac{dt}{e^x} = \frac{dt}{t}$$

$$= \int \frac{t^2-2}{t+1}\,\frac{dt}{t} = \int \frac{t^2-2}{t^2+t}\,dt = \int 1 + \frac{-t-2}{t(t+1)}\,dt =$$

$$\frac{-t-2}{t(t+1)} = \frac{a}{t} + \frac{b}{t+1}$$

$$-t-2 = a(t+1) + bt =$$
$$= at + a + bt =$$
$$= t(a+b) + a$$

$a+b = -1$
$a = -2$
$b = 1$

$$t^2 - 2 : t^2 + t = 1$$
$$-(t^2+t)$$
ost.: $-t-2$

$$= t + \int \frac{-t-2}{t(t+1)}\,dt =$$

$$= t + \int\left(\frac{-2}{t} + \frac{1}{t+1}\right)dt =$$

$$= t - 2\ln|t| + \ln|t+1| - C = e^x - 2x + \ln(e^x+1) - C$$

$$\int \frac{x+3}{(x-1)(x^2-2x+5)}\,dx =$$

$$\frac{x+3}{(x-1)(x^2-2x+5)} = \frac{a}{x-1} + \frac{bx+c}{x^2-2x+5}$$

$$x+3 = a(x^2-2x+5) + (bx+c)(x-1)$$

$$x+3 = ax^2 - 2ax + 5a + bx^2 - bx + cx - c$$

$$x+3 = (a+b)x^2 + (c-b-2a)x + (5a-c)$$

$$a+b=0$$
$$c-b-2a=1$$
$$5a-c=3$$

...

$$a=1$$
$$c=2$$
$$b=-1$$

$$=\int\left(\frac{1}{x-1}+\frac{-x+2}{x^2-2x+5}\right)dx=$$

$$=\int\left(\frac{1}{x-1}+\frac{-x+2}{(x-1)^2+4}\right)dx=$$

$$=\int\left(\left(\frac{1}{x-1}\right)+\frac{-x+2}{4\left(\left(\frac{x-1}{2}\right)^2+1\right)}\right)dx$$

$$=\ln|x-1|+\int\frac{-x+2}{4\left(\left(\frac{x-1}{2}\right)^2+1\right)}dx=$$

$t=\frac{x-1}{2}$ $\quad x=2t+1$

$dt=\frac{1}{2}dx$ $\quad dx=2dt$

$$=\ln|x-1|+\int\frac{-2t+1}{4(t^2+1)}dt = \ln|x-1| + -\int\frac{t}{t^2+1}dt+\frac{1}{2}\arctan t=$$

$u=t^2+1$

$du=2t\,dt$

$$=\ln|x-1|+\frac{1}{2}\arctan t-\int\frac{1}{2u}du=$$

$$=\ln|x-1|+\frac{1}{2}\arctan t-\frac{1}{2}\ln|u|=$$

$$=\ln|x-1|+\frac{1}{2}\arctan\left(\frac{x-1}{2}\right)-\frac{1}{2}\ln\left|\frac{x^2-2x+5}{4}\right|-C=$$

$$=\ln|x-1|+\frac{1}{2}\arctan\left(\frac{x-1}{2}\right)-\frac{1}{2}\ln(x^2-2x+5)-D$$

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SNOV: trig integer

$$\int \cos^m x \sin^n x\,dx \qquad m,n\in\mathbb{N}_0$$

če je m lih: $m=2k+1$

"never keep an odd trig exponent"

$$\cos^m x=\cos x(1-\sin^2 x)^k$$

$$t=\sin x$$

če je n lih: (simetrično)

oba soda:

$$\cos^2 x+\sin^2 x=1-\cos 2x$$

$$\cos^2 x=\frac{1+\cos 2x}{2}$$

$$\sin^2 x=\frac{1-\sin 2x}{2}$$

---

N

$$\int\cos^3 x\sin^4 x\,dx=\int(1-t^2)^2\cdot\cancel{\cos x}\,t^4\frac{dt}{\cancel{\cos x}}=$$

$t=\sin x$

$dt=\cos x\,dx$

$dx=\frac{dt}{\cos x}$

$$=\int(1-t^2)^2t^4\,dt=$$

$$=\frac{t^5}{5}-\frac{t^7}{7}-C=$$

$$=\frac{\sin^5 x}{5}-\frac{\sin^7 x}{7}-C$$

N

$\int \cos^2 x \sin^2 x\,dx = \int \frac{1+\cos(2x)}{2}\cdot\frac{1-\cos(2x)}{2}\,dx =$

$= \frac{1}{4} \int \left(1-\cos^2(2x)\right)dx = \frac{1}{4}\int \sin^2 2x\,dx = \frac{1}{4} =$

$= \dots = \frac{1}{8} x - \frac{1}{32}\sin(4x) - C$

N

$\int \sqrt{1-x^2}\,dx \Rightarrow \int \sqrt{1-\sin^2 t}\,\cos t\,dt = \int \sqrt{\cos^2 t}\,\cos t\,dt =$

$= \int |\cos t| \cos t\,dt =$

$\downarrow$

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$x = \sin t \qquad dx = \cos t\,dt$

$t = \arcsin x$

(zaradi $D_f$ $\sqrt{\ }$)

$= \int \cos^2 t\,dt = \int \frac{1+\cos 2t}{2}\,dt =$

$= \frac{1}{2}\left(t + \frac{1}{2}\sin 2t\right) - C =$

$= \frac{1}{2}\left(\arcsin x + \sin(\arcsin x)\cos(t)\right) - C =$

$= \frac{1}{2}\left(\arcsin x + x\sqrt{1-x^2}\right) - C$

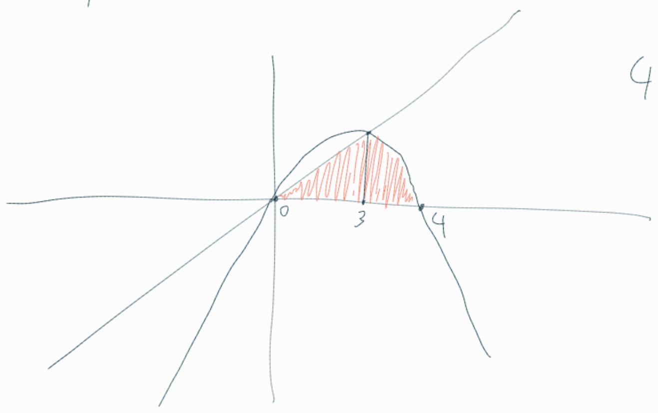
Določeni integral <== snov

$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$

N

Izračunaj ploščino lika, ki ga omejujejo abscisna os in grafa funkcije

$f(x) = 4x - x^2$ ter $g(x) = x$




$4x - x^2 = x$

$x^2 - 3x = 0$

$x(x-3)$

$x_1 = 0$

$x_2 = 3$

$pl = \int_0^3 x\,dx + \int_3^4 4x - x^2\,dx = 4{,}5 + \left(2x^2 - \frac{x^3}{3}\right)\Big|_3^4 =$

$= \frac{9}{2} + 32 - \frac{64}{3} - 18 + 9 = \dots = 34/6 \cdot 1^2$

N

ploščina med grafoma:

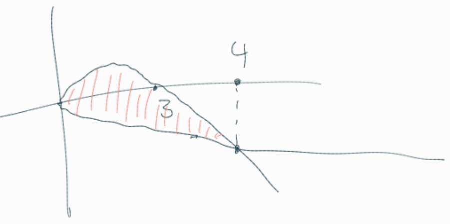
$f(x) = \frac{(3-x)\sqrt{x}}{3} \qquad g(x) = -\frac{\sqrt{x}}{3}$

$pl = \int_0^4 \left(f(x) - g(x)\right)dx =$

$\frac{1}{3}\int_0^4 \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right)dx =$

$= \frac{1}{3}\left(\frac{8}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right)\Big|_0^4 =$

$= \frac{1}{3}\cdot\frac{8}{3}4^{\frac{3}{2}} - \frac{2}{5}\cdot 4^{\frac{5}{2}} - 0 = \dots = \frac{128}{45}$




$\frac{(3-x)\sqrt{x}}{\cancel{3}} = -\frac{\sqrt{x}}{\cancel{3}}$

$\sqrt{x}(4-x) = 0$

$x_1 = 0$

$x_2 = 4$