L'Hôpitalovo pravilo

$\lim\limits_{x\to a} \frac{f(x)}{g(x)}$; $\bullet \lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) \in \{0, \pm\infty\}$

$a \in \mathbb{R} / \infty / -\infty$ $\quad \bullet \exists \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$

$\uparrow, \to, \downarrow$

$\Rightarrow \lim\limits_{x\to a} \frac{f(x)}{g(x)} = \lim\limits_{x\to a} \frac{f'(x)}{g'(x)}$

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$$\lim_{x\to\infty} \frac{x^n}{e^x} = \lim_{x\to\infty} \frac{n x^{n-1}}{e^x} \underset{n\text{-krat}}{=} \dots = \lim_{x\to\infty} \frac{n!\, x^0}{e^x} = \lim_{x\to\infty} \frac{n!}{e^x} = 0$$

$$\lim_{x\to\infty} \left(e^x + x\right)^{1/x} = \dots$$

$$\lim_{x\to 0} \frac{\frac{1}{\cos^2 x} - \cos x}{1 - \cos x} = \lim_{x\to 0} \frac{\cancel{\frac{2\cos x \sin x}{\cos^4 x}} + \sin x}{\sin x} = \lim_{x\to 0} \frac{2}{\cos^3 x} + 1 = 2 + 1 = 3$$

10 11 12 3 9 8 7 6 5 4 $\quad Z_{11}$ ura

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TAYLORJEVA VRSTA

če je f $(n+1)$ krat odvedljiva na odp. int. I, za $a \in I$ velja

$$\forall x \in I \ \exists \xi \in (a, x) \ni: f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k + R_{n,f,a}(x)$$

$$R_{n,f,a}(x) = \frac{(x-a)^{n+1} \cdot f^{(n+1)}(\xi)}{(n+1)!}$$

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let $p(x) = 2x^3 - 5x^2 + x + 7$

Določi Taylorjev polinom reda 3 v okolici 1:

$$T_{3,1,f}(x) = \sum_{k=0}^{3} \frac{p^{(k)}(1)}{k!}(x-1)^k$$

potrebujemo:

$p^0(1) = 2 - 5 + 1 + 7 = 5$

$p'(1) \cdot p'(x) = 6x^2 - 10x + 1 \quad p'(1) = -3$

$p^2(1) \cdot p''(x) = 12x - 10 \quad p''(1) = 2$

$p^3(1) \cdot p'''(x) = 12 \quad p'''(1) = 12$

$$T_{3,1,f}(x) = 5 + -3(x-1) + (x-1)^2 + 2(x-1)^3 = p(x)$$

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let $f \in C^3$ in $f \notin C^4$ in $f(\frac{\pi}{2}) = \pi$ in $\sin x + \sin f(x) = 1 \quad \forall x \in \mathbb{R}$

Določi $T_{f,\underset{\text{red}}{3},\underset{\text{točka}}{\frac{\pi}{2}}}(x)$

$\sin f(x) = 1 - \sin x \quad /'()$

$f'(x) \cos f(x) = -\cos x$

$f'(x) = \frac{-\cos x}{\cos f(x)}$

$f'(\frac{\pi}{2}) = \frac{-\cos\frac{\pi}{2}}{\cos \pi} = \frac{-0}{-1} = 0$

$f'(x) \cos f(x) = -\cos x \quad /'()$

$f''(x)\cos f(x) - \left(f'(x)\sin f(x)\right) f'(x) = \sin x$

$f''(x) = \frac{\sin x}{\cos f(x)}$

$f''(\frac{\pi}{2}) = \frac{\sin\frac{\pi}{2}}{\cos \pi} = \frac{1}{-1} = -1$

$f''(x)\cos f(x) = 0 \quad /'()$

$f'''(x)\cos f(x) - \left(f''(x)\sin f(x)\right) f'(x) = 0$

$f'''(x) = -\cos f(x)$

$f'''(\frac{\pi}{2}) = -\cos\pi = 1$

PRAVILNO JE: $T(x) = \pi - \frac{1}{2} \cdot (x - \frac{\pi}{2})^2$

koef: $\pi, 0, -1, 0$

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a.) $f(x)=\arctan(x)$ razvij v $T_{f,0}(x)$.
točka

b.) izračunaj limito
$$\lim_{x\to0}\frac{\arctan x - x}{x^3}$$

c.) izračunaj $f^{(n)}(0)$ $\forall n\in\mathbb{N}$.

$$f(x)=\sum_{n=0}^{\infty}(-1)^n x^{2n+1}$$

b.) $\lim_{x\to0}\frac{\arctan x - x}{x^3}$ vstavimo T in poenostavimo v $-\frac{1}{3}$.

c.) $$\frac{f^{(n)}(0)}{n!}=\begin{cases}0 & ; n=2k\\ \frac{1}{(-1)^{\frac{n-1}{2}}n} & ; n=2k+1\end{cases}\quad k\in\mathbb{Z}$$

$$f^{(n)}(0)=\begin{cases}0 & ; n=2k\\ \frac{n!}{(-1)^{\frac{n-1}{2}}n} & ; n=2k+1\end{cases}\quad k\in\mathbb{Z}$$

$$\| \quad \frac{(n-1)!}{(-1)^{\frac{n-1}{2}}}$$

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Razvij f v Taylorjevo vrsto okoli točke a.

a.) $f(x)=\frac{1}{x}$ $a=1$.

razvijamo okoli točke, ki ni 0.
zato uvedemo novo var. t $\boxed{t=x-a}=x-1$

$$f(x)=\frac{1}{x}=\frac{1}{t+1}=\sum_{k=0}^{\infty}(-1)^k t^k=\sum_{k=0}^{\infty}(-1)^k(x-1)^k$$

b.) $f(x)=\frac{1}{x^2+5x+6}$ $a=1$

let $\boxed{t=x-a}=x-1$

$$f(x)=\frac{1}{(t+1)^2+5(t+1)+6}=\frac{1}{t^2-3t+2}=\frac{1}{(t-1)(t-2)}$$

$$=\frac{A}{t-1}+\frac{B}{t-2}=\frac{-1}{t-1}+\frac{1}{t-2}=\frac{1}{1-t}+\frac{1}{t-2}=\sum_{n=0}^{\infty}t^n-\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{t}{2}\right)^n=$$

$A(t+1)=1$ ... $A=-1$
$B(t-1)=1$ ... $B=1$

$$=\sum_{n=0}^{\infty}t^n\left(1-\frac{1}{2^{n+1}}\right)=\sum_{n=0}^{\infty}(x-1)^n\left(1-\frac{1}{2^{n+1}}\right)$$

c.) $f(x)=\sin^2 x$ $a=0$

funkcionalnost

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Funkcionalno

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$$f(x) = \ln \frac{\sqrt{x^2+1}}{2x^2+1} = \qquad a = 0$$

$$= \frac{1}{2}\ln(x^2+1) - (2x^2+1) = \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^{2n} - \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (2x^2)^n =$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^{2n} \left(\frac{1}{2} - 2^n\right)$$

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$$f(x) = \sinh(x) = \qquad a = 0$$

$$= \frac{e^x - e^{-x}}{2} = \frac{1}{2}(e^x - e^{-x}) = \frac{1}{2}\left(\sum_{n=0}^{\infty} \frac{x^n + (-x)^n}{n!}\right) = \sum_{n=0}^{\infty} \frac{x^n + (-x)^n}{2n!} =$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!} \underbrace{\left(1 - (-1)^n\right)}_{0 \text{ za sode } n} = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$$